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Department of Mathematics

NC STATE UNIVERSITY

MA 305 Intro Elem Lin Algebra, first mid-semester examination, Feb 13, 2001

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Your Name: ______________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of **46 points**. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	____
2	____
3	____
Total	____

If you are taking the exam later, please sign the following statement:

I, ________________*, affirm that I have no knowledge of the contents of this exam.*

____
Signature

Problem 1 (19 points)

(a, 3 pts) Please name a well-known mathematician who was born in the USA.

(b, 4 pts) In how many multipications of pairs of integers can one compute $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{256}$? Please explain.

(c, 4 pts) By laws that the transposition and inversion operations for matrices satisfy please prove that for an invertible matrix A we have $(A^T)^{-1} = (A^{-1})^T$.

(d, 4 pts) Suppose that in a Maple session the variable `A` has been assigned a 4×4 invertible matrix. If you then execute the Maple commands `with(linalg): rref(A);` what answer to you expect?

(e, 4 pts) A binary operation on an arbitrary set S may or may not satisfy the mathematical laws of (i) associativity, (ii) S has a unit element, (iii) each element in S has an inverse element, and (iv) commutativity. For the concrete set $S = \mathbb{R}^{n \times n}$ and matrix multiplication as the binary operation, which of these four laws are satisfied?

Problem 2 (15 points): Consider the following augmented matrix of a system of linear equations, where the first 3 columns correspond to the variables x, y, z and where a, b, and c are real parameters.

$$\begin{bmatrix} 1 & 1 & -1 & \vdots & 0 \\ -a & 1 & -1 & \vdots & (1+a)c \\ 0 & b & -b & \vdots & c \end{bmatrix} \tag{1}$$

(a, 4pts) For which values of a, b, and c is the matrix (1) in row echelon form? Please give all the conditions.

(b, 6pts) By performing Gaussian elimination on the cases $a = -1$ and $a \neq -1$ separately, determine for which values of a, b, and c the linear system corresponding to the augmented matrix (1) is consistent. For each condition please state the row-echelon form.

(c, 5pts) For each of the conditions discovered in part b, please perform the back-substitution to solve the system.

Problem 3 (12 points): Consider the following matrix A together with its factorization into elementary matrices. Here α is a non-zero real parameter, and β is a real parameter.

$$\begin{bmatrix} 1 & 0 & 0 \\ \beta & 0 & 1 \\ 0 & 1/\alpha & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\alpha \end{bmatrix}}_{E_1} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{E_2} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{bmatrix}}_{E_3}$$

This problem computes A^{-1} as $E_3^{-1} \cdot E_2^{-1} \cdot E_1^{-1}$. Please perform the following steps.

(a, 4 pts) Please write E_1, E_2, E_3 as elementary matrices $E_I(\ldots)$ or $E_{II}(\ldots)$ or $E_{III}(\ldots)$.

(b, 4 pts) Please write $E_3^{-1}, E_2^{-1}, E_1^{-1}$ as elementary matrices $E_I(\ldots)$ or $E_{II}(\ldots)$ or $E_{III}(\ldots)$.

(c, 4 pts) Please write out E_1^{-1} as a matrix. Then compute the product $E_2^{-1} \cdot E_1^{-1}$ by performing the elementary row operation for E_2^{-1} determined in part b. Then compute the product $E_3^{-1} \cdot (E_2^{-1} E_1^{-1})$, again by performing the elementary row operation for E_3^{-1} of part b.