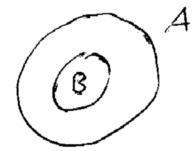
REAL ANALYSIS

Sec. 1.1 → Notation / Set Theory

SET : collection of objects. Let A be a set



$x \in A$

$x \notin A$

every element of B is contained in A

SUBSETS $B \subseteq A$, PROPER SUBSET $B \subset A$

every element of B is in A, but there is at least one element of A that is not in B

Exa. $A = \mathbb{N}$ $B = 2\mathbb{N}$ = even numbers

$x = 2$ $x \in A$,

$y = \sqrt{2}$ $y \notin A$ $\mathbb{N} = \{1, 2, 3, \dots\}$

$B \subseteq A$, $B \subset A$, $\mathbb{R} \supset A$

Sets A, B $A = B \iff A \subseteq B$ and $B \subseteq A$

To define a set, we use the notation

$$B = \{x \in A : P(x)\}$$

the elements of B are those elements of A satisfying the property $P(x)$.

Exa.

$$B = \{x \in \mathbb{N} : x^2 - 3x + 2 = 0\}$$

B is the set of all $x \in \mathbb{N}$ satisfying the equation, namely

$B = \{1, 2\}$

Ex. Define using set notation the even and odd integers:

$E = \{n \in \mathbb{N} : n = 2k, k \in \mathbb{N}\}$ or $\{2k : k \in \mathbb{N}\}$

$O = \{n \in \mathbb{N} : n = 2k-1, k \in \mathbb{N}$ $\{2k-1 : k \in \mathbb{N}\}$

SET OPERATIONS A, B sets

$A \cup B = \{x : x \in A$ or $x \in B\}$ UNION

$A \cap B = \{x : x \in A$ and $x \in B\}$ INTERSECTION

$A \setminus B = \{x : x \in A$ and $x \notin B\}$ COMPLEMENT

$\emptyset$ EMPTY SET = set with no elements

If $A \cap B = \emptyset$, then A, B are DISJOINT

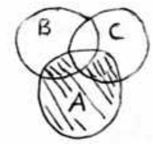
$\bigcup_{n=1}^{\infty} A_n = \{x : x \in A_n$, for some $n \in \mathbb{N}\}$ $\{A_n : n \in \mathbb{N}\}$ is a countable collection of sets

$\bigcap^{\infty} A_n = \{x : x \in A_n$ for all $n \in \mathbb{N}\}$

Thm Let A, B, C be sets

(a) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

(b) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$



PROOF (b) Need to show that

(1) $A \setminus (B \cap C) \subseteq (A \setminus B) \cup (A \setminus C)$

(2) $(A \setminus B) \cup (A \setminus C) \subseteq A \setminus (B \cap C)$

(1) If $x \in A \setminus (B \cap C)$, then $x \in A$ but $x \notin B \cap C$. Hence $x \in A$ but ~~not~~ $x \notin B$ or $x \in A$ but $x \notin C$. Thus $x \in (A \setminus B) \cup (A \setminus C)$

(2) If $x \in A$ but $x \notin B$ or $x \in A$ but $x \notin C$, then $x \in A$ but x cannot belong to both B and C. Thus $x \in A \setminus (B \cap C)$.

DEF CARTESIAN PRODUCT OF SETS Let A, B be nonempty sets.

$$A \times B = \{ (a, b) : a \in A, b \in B \}$$

Notice: this is a collection of pairs. Order is important!

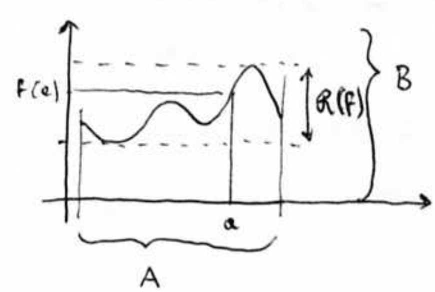
Ex $A = \mathbb{N}$, $B = \{0, 1\}$

$$A \times B = \{ (n, 0), (n, 1) : n \in \mathbb{N} \}$$

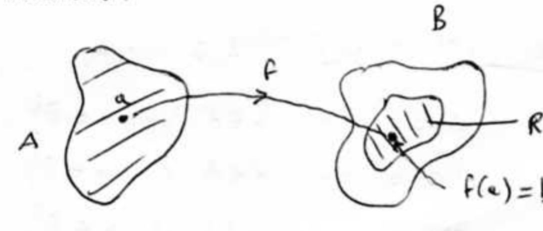
DEF FUNCTION For A, B sets, ~~the~~ a function f from A to B $(f : A \to B)$ is a set of ordered pairs $(a, b) \in A \times B$ such that for each $a \in A$ there is a unique $b \in B$ s.t. $b = f(a)$

A is the DOMAIN of $f = \mathcal{D}(f)$ $\qquad A = \mathcal{D}(f)$

$R(f) = \{ b \in D : b = f(a), a \in A \} \subseteq B$ $\qquad R(f) \subseteq B$



Set NOTATION:



$a = f^{-1}(b)$ INVERSE IMAGE

CAN YOU INVERT f?

You need to be sure that to any $b \in R(f)$ it corresponds a unique $a \in A$ s.t. $f(a) = b$. $\to$ BIJECTIVE FUNCTION