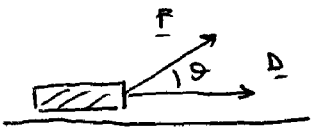
SOLUTION

Math 242 Sec. 12 – Fall 2006 Name:

TEST #1

No calculators, books or notes allowed. Please, justify your answers and write clearly if you want credit for your work.

(1) [2 Pts.] A wagon is pulled a distance of 20 m along a horizontal path by a constant force of 35 N. The handle of the wagon is held at an angle of 30 degrees above the horizontal. How much work is done? (Recall: $\sin(\pi/6) = 1/2$; $\cos(\pi/6) = \sqrt{3}/2$.)



$W = \underline{F} \cdot \underline{D} = |\underline{F}|\,|\underline{D}| \cos\theta$ (1Pt)

$= 35\ 20 \cos\frac{\pi}{6} = \boxed{350\sqrt{3}}$ (1Pt)

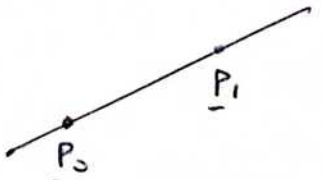
(2) [2 Pts.] Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (6, 3, -1)$, $\mathbf{v} = (0, 1, 2)$, and $\mathbf{r} = (4, -2, 5)$.

VOLUME $= \underline{u} \times \underline{v} \cdot \underline{r}$

$$= \begin{vmatrix} 6 & 3 & -1 \\ 0 & 1 & 2 \\ 4 & -2 & 5 \end{vmatrix}$$ (1Pt)

$= 6 \cdot 9 + 3 \cdot 8 + 4 = \boxed{82}$ (1Pt)

(3) [3 Pts.] Find the parametric equation of the line containing the points $P_0(1,-2,0)$ and $P_1(-3,2,2)$.



$\overrightarrow{P_0P_1} = (-4, 4, 2) = \underline{v}$ (1Pt)

Choose $P_0(1,-2,0)$ (1Pt)

$$\begin{cases} x(t) = 1 - 4t \\ y(t) = -2 + 4t \\ z(t) = 2t \end{cases}$$ (1Pt)

(4) [4 Pts.] Find the distance between the two parallel planes $3x+6y-9z=4$ and $x+2y-3z=1$.

plane (1) $3x+6y-9z=4$

Find 1 point on plane (1) $P_1\left(\frac{4}{3},0,0\right)$ (1Pt)

plane (2) $x+2y-3z=1 \iff x+2y-3z-1=0$ (1Pt)

distance from plane (2) to P_1:

$$D = \frac{\frac{4}{3}-1}{\sqrt{1+4+9}} = \frac{1/3}{\sqrt{14}} = \frac{1}{\sqrt{126}}$$ (2Pt)

(5) [4 Pts.] Find the equation of the plane that contains the line $x=-1-2t, y=t, z=5+3t$ and is perpendicular to the plane $3x-y+8z=17$.

plane (1) $3x-y+8z=17$ $\underline{n} = (3,-1,8)$

line $\underline{r}(t) = (1-2t, t, 5+3t)$ $\underline{v} = (-2,1,3)$

The unknown plane has normal vector:

$$\underline{N} = \underline{v} \times \underline{n} = (-2,1,3) \times (3,-1,8)$$
$$= (11, 25, -1)$$ (2Pt)

Choose 1 Point on line

$P_0(-1,0,5)$ (1Pt)

Eq. of plane: $11(x+1) + 25y - (z-5) = 0$

$$11x + 25y - z + 16 = 0$$ (1Pt)