

April 2006 M⁶ Problem

Solution

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Problem: A study shows that the percentage of left-handed people among mathematicians is greater than the percentage of left-handed people among non-mathematicians. Prove that it follows that the percentage of mathematicians among left-handed people is greater than the percentage of mathematicians among right-handed people.

Solution:

We can divide the entire population into four groups based on whether they are left-handed or right-handed, a mathematician, or a non-mathematician. We label these four groups as A, B, C, D with these numbers representing the whole number of individuals with the given pair characteristics.

	Mathematicians	Non-Mathematicians
Right-Handed	D	C
Left-Handed	A	B

We next examine the statement “the percentage of left-handed people among mathematicians is greater than the percentage of left-handed people among non-mathematicians.” In terms of the table above this means:

$$\frac{A}{A+D} > \frac{B}{B+C} \quad (1)$$

And we want to use (1) to imply:

$$\frac{A}{A+B} > \frac{D}{C+D} \quad (2)$$

Starting at (1) we can easily get to (2) by simple arithmetic (We note here that $A, B, C,$ and D are all non-negative integers - it stands to reason that there is at

least one person in each group).

$$\frac{A}{A+D} > \frac{B}{B+C} \quad (3)$$

$$A(B+C) > B(A+D) \quad (4)$$

$$AB+AC > AB+BD \quad (5)$$

$$AC > BD \quad (6)$$

$$AC+AD > BD+AD \quad (7)$$

$$A(C+D) > D(A+B) \quad (8)$$

$$\frac{A}{A+B} > \frac{D}{C+D} \quad \square \quad (9)$$