Solution to the November 2007 M^6 Problem

Problem: Is it possible that in a group of 35 people each person knows exactly 11 others? Justify your answer.

Answer: This is impossible.

Solution: Let each pair of people who know each other hold a jumping rope. If each person knows exactly 11 people, then each person will hold exactly 11 jumping rope ends. So all 35 people hold 11×35 jumping rope ends, which is an odd number. However, each jumping rope has two ends, so the total number of jumping rope ends must be even. Contradiction.

Discussion: The same argument will lead to a more general statement: *In a group of an odd number of people it is impossible that each person knows exactly the same odd number of people.*

The argument can be also put in the context of the graph theory. Each person can be represented by a vertex. Two vertices are connected by an edge if the corresponding persons know each other. The degree of a vertex is the number of edges emitted from this vertex. It is equal to the number of people the person represented by this vertex knows. Obviously, the sum of the degrees of all vertices is twice the number of edges and, therefore, is even.