Claim: There are infinitely many prime numbers.

Proof. Suppose, to the contrary, that there are a finite number of prime numbers. Let p_1, p_2, p_3,…, p_k be the entire collection of prime numbers. Define $n = (p_1p_2...p_k) + 1$, that is, n is equal to the product of every prime number plus one. The n is a natural number, and by the Fundamental Theorem of Algebra, n can be expressed uniquely as a product of primes. That implies that n has a prime divisor q. Then $q > 1$. And, since q is prime, q is in the collection of all primes, and therefore, q divides the product $(p_1p_2...p_k)$. Since q divides both n and the product of all the primes, it divides their difference, i.e., $n - (p_1p_2...p_k) = 1$. But the only divisor of 1 is 1. So $q = 1$. Then we have $q = 1$ and $q > 1$, which is a contradiction. We conclude that our supposition that the primes were finite in number is false, and therefore, there must exist infinitely many prime numbers.

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