November 2006 M^6 Problem

Let f be a univariate polynomial with complex coefficients, that is, $f \in \mathbb{C}[x]$. Let also f have only rational values at all rational points, that is, $f(q) \in \mathbb{Q}$ for all $q \in \mathbb{Q}$. Does f always have rational coefficients? Justify your answer.

Solutions can be submitted to Alexey Ovchinnikov, `aiovchin@ncsu.edu`, starting from noon November 9 until midnight November 30, 2006. Please, make sure that you have read the contest rules: e-mail attachments are limited to 200Kb.