FEBRUARY 2008 M^6 PROBLEM

JAMES HOLBERT
JMHOLBER@NCSU.EDU
SENIOR MATH MAJOR

One in seven mathematicians is a philosopher, while one in nine philosophers is a mathematician. Is the total number of mathematicians greater than the total number of philosophers, or conversely?

Solution. There are more philosophers. Let Φ be the set of all philosophers and M the set of all mathematicians. Set the sample space $\Sigma = \Phi \cup M$. Let φ denote the event a person is a philosopher, μ the event a person is a mathematician, and $\varphi\mu$ the event a person is both a mathematician and philosopher. We are given the conditional probabilities $P(\varphi|\mu) = \frac{1}{7}$ and $P(\mu|\varphi) = \frac{1}{9}$. We seek the relation between $|\Phi|$ and $|M|$.

Note $P(\varphi) = \frac{|\Phi|}{|\Sigma|}$ and $P(\mu) = \frac{|M|}{|\Sigma|}$. Since $P(\varphi|\mu) = \frac{P(\varphi\mu)}{P(\mu)}$ and $P(\mu|\varphi) = \frac{P(\varphi\mu)}{P(\varphi)}$, we have $\frac{1}{7} = \frac{|\Sigma|}{|M|}P(\varphi\mu)$ and $\frac{1}{9} = \frac{|\Sigma|}{|\Phi|}P(\varphi\mu)$. Thus $|M| = 7\,|\Sigma|\,P(\varphi\mu)$ and $|\Phi| = 9\,|\Sigma|\,P(\varphi\mu)$. Note $|\Sigma| > 0$ and $P(\varphi\mu) > 0$. Therefore $|M| < |\Phi|$. □

Date: 19 February 2008.