

Kelsey Hawkins
KPHawkins@gmail.com
Junior (2nd year)
Majoring in Applied Mathematics and Computer Science.

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Since $f(x) - x$ and $f(x)$ are polynomials then $g(x) = f(x) - x$ and $f(x)$ are real and defined for all x in the real numbers. Since $g(x)$ does not have any real roots, there does not exist an x in the real numbers such that $g(x) = 0$. Suppose there exists a k such that $f(f(k)) - k = 0$ and take $m = f(k)$. Thus, since $f(x)$ is real and defined for the real line, m is real and $f(m) = k$. Therefore, $g(m) = f(m) - m = k - f(k) = -(f(k) - k)$. But $g(k) = f(k) - k$ so $g(k) = -g(m)$. If $g(k) > 0$ then $g(m) < 0$ and by the Location of Roots Theorem the function $g(x)$ has a root. But $g(x)$ does not have any real roots, so by contradiction, $g(k) \leq 0$. Similarly, if $g(k) < 0$ then $g(x)$ has a root. Thus $g(k) = -g(m) = 0$ which is a root itself and thus we have full contradiction. Therefore, there does not exist a k such that $f(f(k)) - k = 0$ and thus the function $f(f(x)) - x$ has no real roots.