

## Solution to February 2004 M<sup>6</sup> Problem

Let  $x$  be the number sought for. It is given that  $\sqrt[3]{x} = x/1000 - a$ , where  $0 \leq a < 1$ . Denote  $y = \sqrt[3]{x}$ . Then, the previous equality transforms into  $y^3 - 1000y - 1000a = 0$ , or alternatively,  $y^2 = 1000 + 1000a/y$ . This implies  $y^2 \geq 1000$ , i.e.,  $y \geq 32$ . (Remember that both  $x$  and  $y$  are supposed to be integer.) On the other hand,  $y^2 = 1000 + 1000a/y \leq 1000 + 1000/32$  because  $a < 1$  and  $y \geq 32$ . Consequently,  $y \leq 32$  and therefore,  $y = 32$ , which means that  $x = y^3 = 32768$ .