Solution to Wolfpack M6 February Problem

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Problem:
One in seven mathematicians is a philosopher, while one in nine pihilosophers is a mathematician. Is the total number of mathematicians greater than the total number of philosophers, or conversely?

Solution:
Let M denote the event that a random person selected is a mathematician, while K denotes the event that the random person selected is a philosopher.

Using these notations and the common notation for the probability function, $P : S \to [0, 1]$, we rationalize the following:

$$P(K|M) = \frac{1}{7}$$
$$P(M|K) = \frac{1}{9}$$

Using basic conditional probability formulas, we find:

$$P(K|M) = \frac{P(K \cap M)}{P(M)}$$
$$P(K|M) = \frac{P(M|K) \cdot P(K)}{P(M)}$$
$$\implies \frac{1}{7} = \frac{\frac{1}{9} \cdot P(K)}{P(M)}$$
$$\implies \frac{9}{7} = \frac{P(K)}{P(M)}$$

Since $\frac{P(K)}{P(M)} = \frac{9}{7} \implies \frac{P(K)}{P(M)} > 1 \implies P(K) > P(M)$, let m denote the number of mathematicians in our sample space, k denote the number of philsophers, and n the total of number of people in the sample space. Thus $P(K) = \frac{k}{n}$ and $P(M) = \frac{m}{n}$. Thus $P(K) > P(M) \iff \frac{k}{n} > \frac{m}{n} \implies k > m$

$\therefore$ The total number of mathematicians is less than the total number of philosophers.

$\mathbb{Q}.\mathbb{E}.\mathbb{D}.$