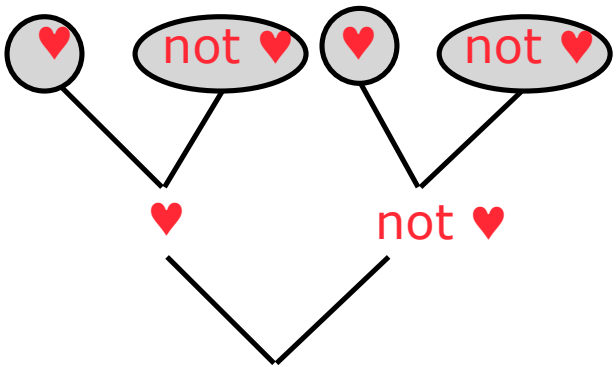
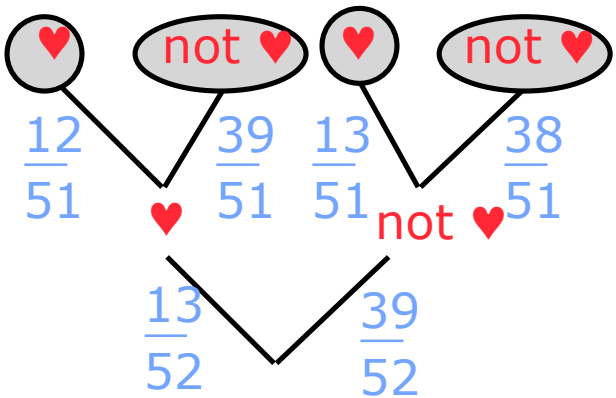
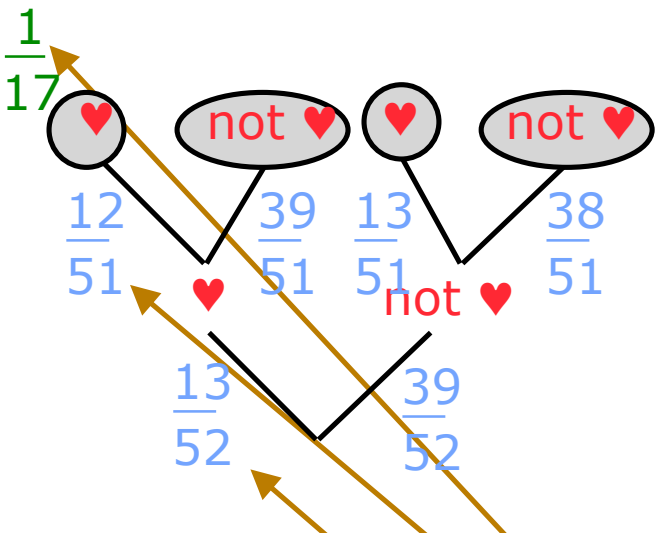
Deal 2 cards from a deck and keep track of whether the cards being dealt are hearts.



Deal 2 cards from a deck and keep track of whether the cards being dealt are hearts.



Deal 2 cards from a deck and keep track of whether the cards being dealt are hearts.



$$P(\text{both hearts}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$$

Deal 2 cards from a deck and keep track of whether the cards being dealt are hearts.

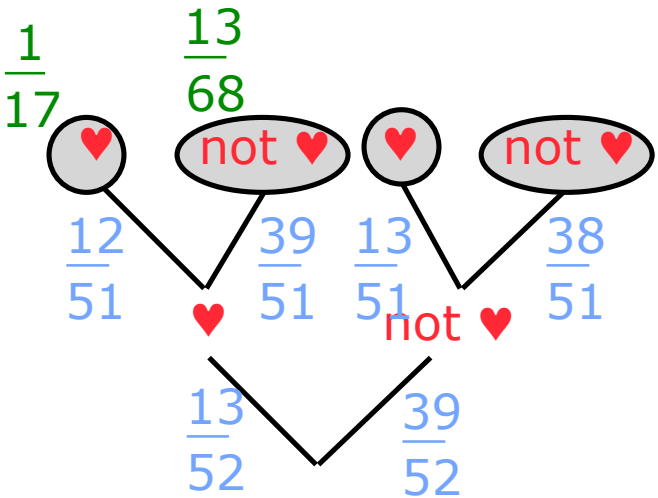
$P(B \cap A) = P(A) \times P(B \mid A)$

P(both ♥'s) =

P(1st card is a ♥) × P(2nd is a ♥ | 1st is a ♥)

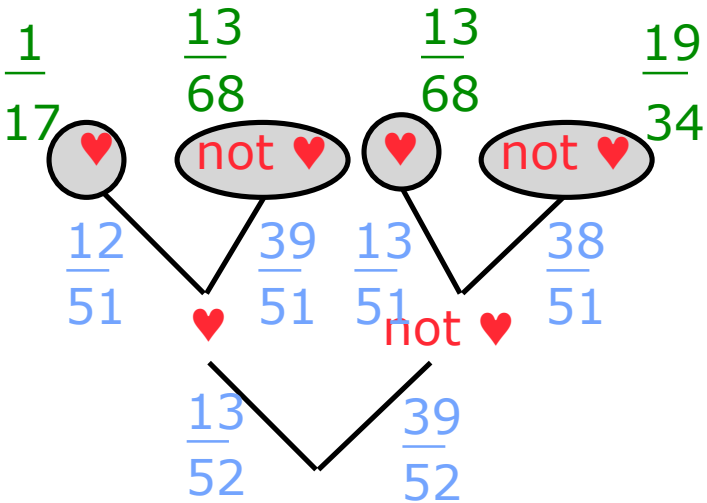
$$P(\text{both hearts}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$$

Deal 2 cards from a deck and keep track of whether the cards being dealt are hearts.



$$P(\text{1st heart and 2nd not a heart}) = \frac{13}{52} \times \frac{39}{51} = \frac{13}{68}$$

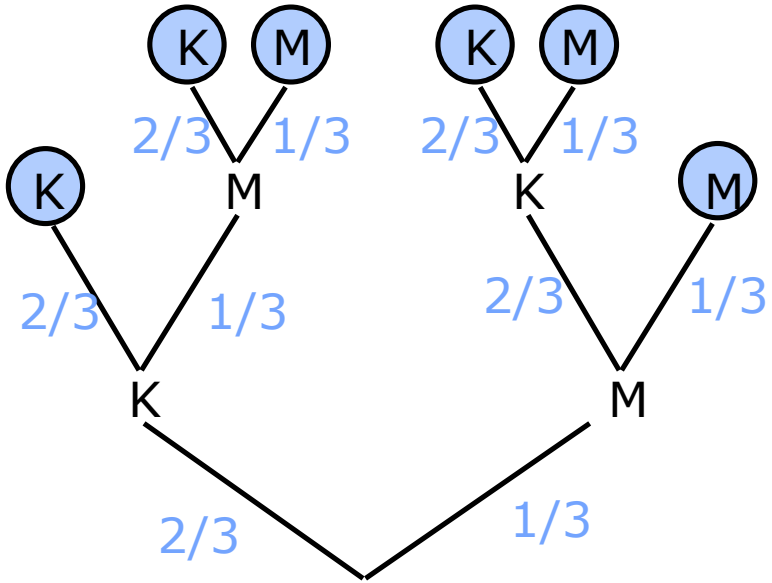
Deal 2 cards from a deck and keep track of whether the cards being dealt are hearts.



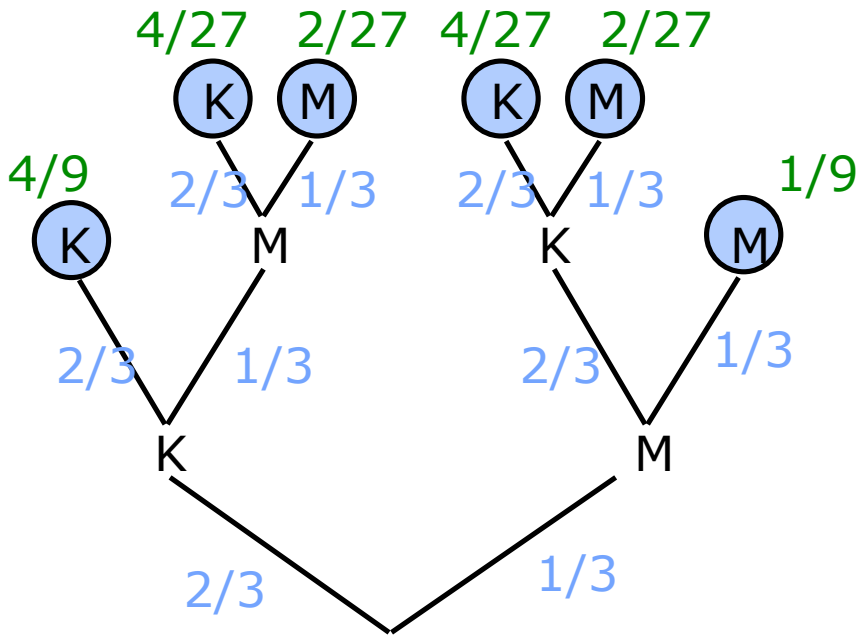
$\frac{39}{52} \times \frac{13}{51} = \frac{13}{68}$

$\frac{39}{52} \times \frac{38}{51} = \frac{19}{34}$

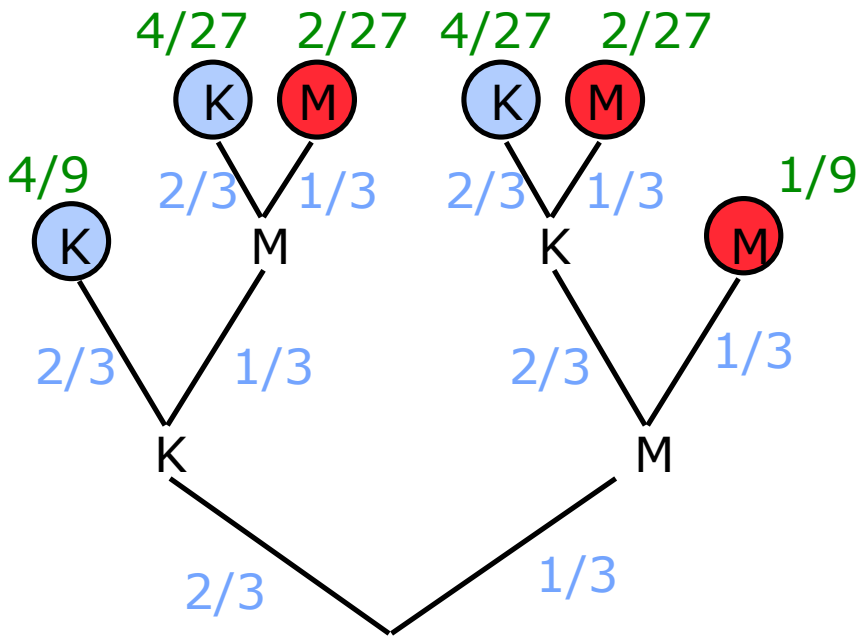
Michelle and Kim are playing a tennis match. The winner will be the first person to win two sets. Kim is the better player, and in fact whenever they play a set the probability is 2/3 that Kim will win the set. What is the probability that Michelle will win the match? If she wins the first set, what then is the probability that she will win the match? What is the probability that she wins the first set given that she wins the match? What is the probability that the match will last for three sets?



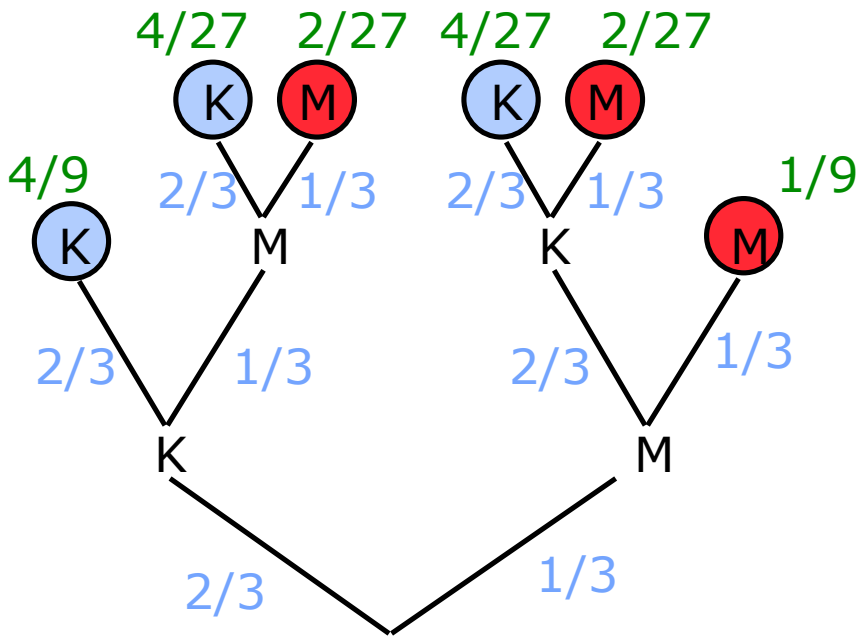
$2/3 \times 1/3 \times 2/3 = 4/27$, etc.



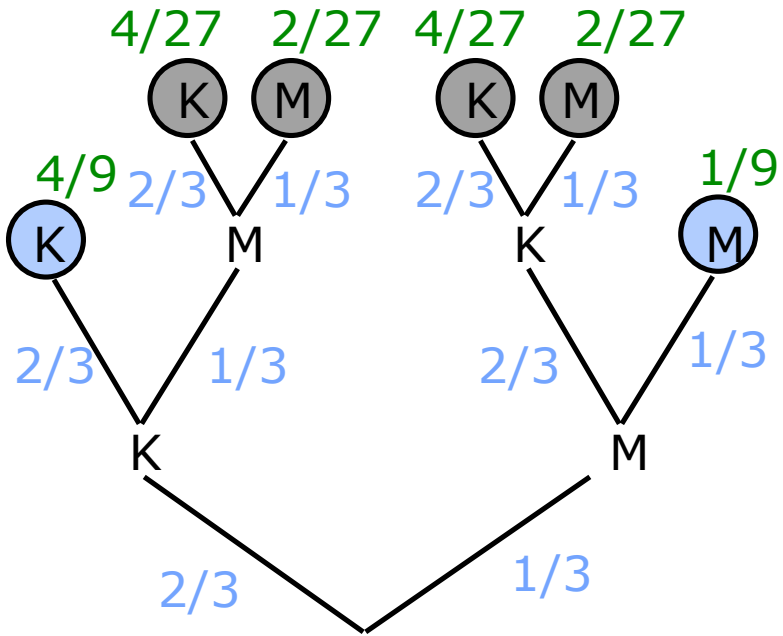
What is the probability that Michelle will win the match?



What is the probability that Michelle will win the match? 2/27 + 2/27 + 1/9 = 7/27



If Michelle wins the first set, what then is the probability that she will win the match? i.e. P(wins match | wins 1st set) = ?

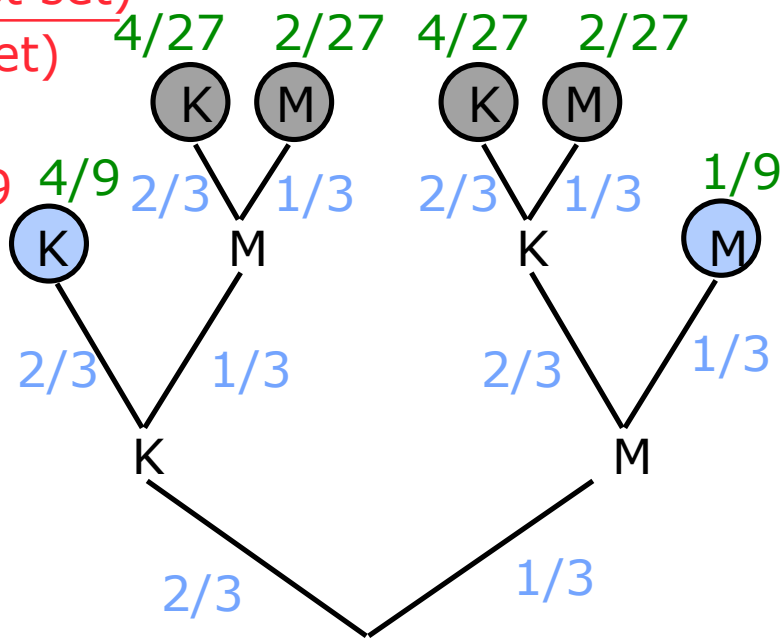


If Michelle wins the first set, what then is the probability that she will win the match? i.e.

$P(\text{wins match} \mid \text{wins 1st set}) =$

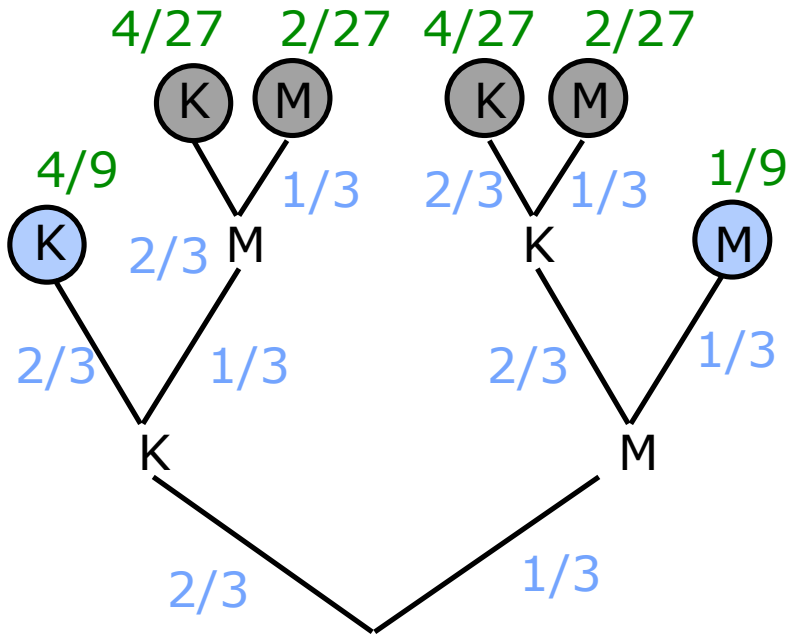
$$= \frac{P(\text{wins match} \,\&\, \text{1st set})}{P(\text{wins 1st set})} = \frac{2/27 + 1/9}{4/27 + 2/27 + 1/9}$$

$$= \frac{5/27}{1/3} = \frac{5}{9}$$



What is the probability Michelle wins the first set given that she wins the match?

i.e. P(wins 1st set | wins match) = ?



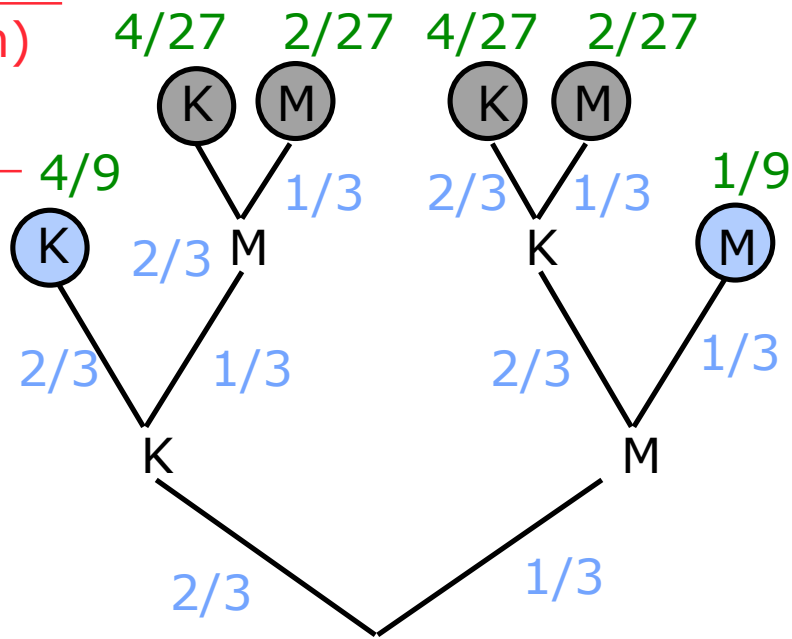
What is the probability Michelle wins the first set given that she wins the match?

i.e. P(wins 1st set | wins match) =

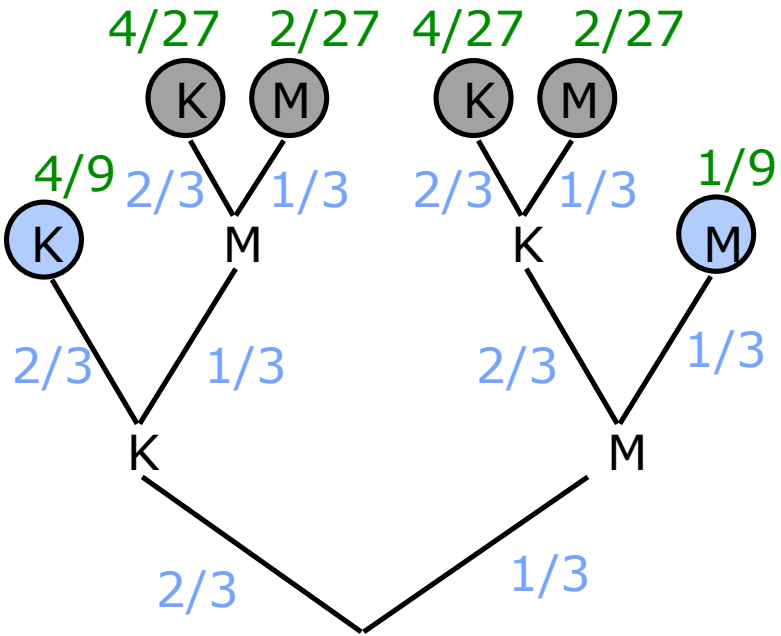
$$\frac{P(\text{wins 1st set \& match})}{P(\text{wins match})}$$

$$= \frac{2/27 + 1/9}{7/27}$$

$$= \frac{5/27}{7/27} = \frac{5}{7}$$

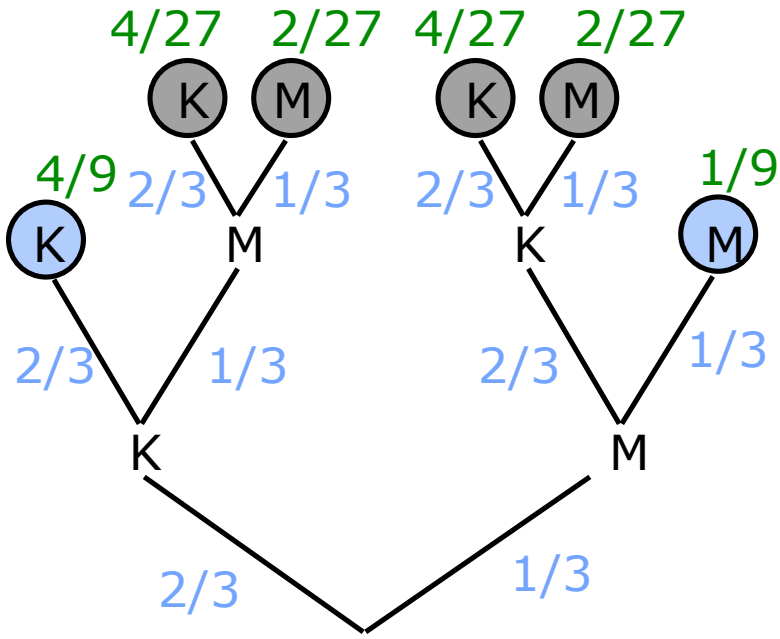


What is the probability the match lasts 3 sets?



What is the probability the match lasts 3 sets?

4/27 + 2/27 + 4/27 + 2/27 = 12/27 = 4/9



Picking a t-shirt

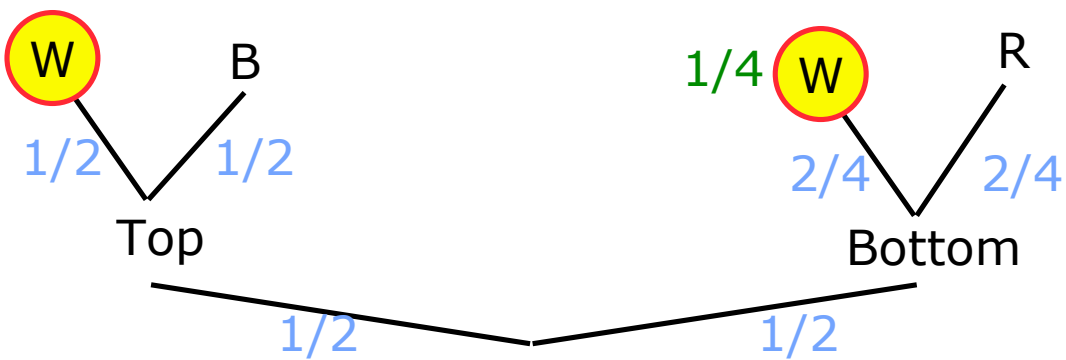
Aaron has 2 drawers containing t-shirts. The top drawer has 1 white t-shirt and 1 blue t-shirt. The bottom drawer has 2 white t-shirts and 2 red t-shirts. He selects a drawer at random and pulls out t-shirts one at a time (without replacement) until he has a white t-shirt.

(a) Draw a tree diagram for this process (including probabilities in the tree).

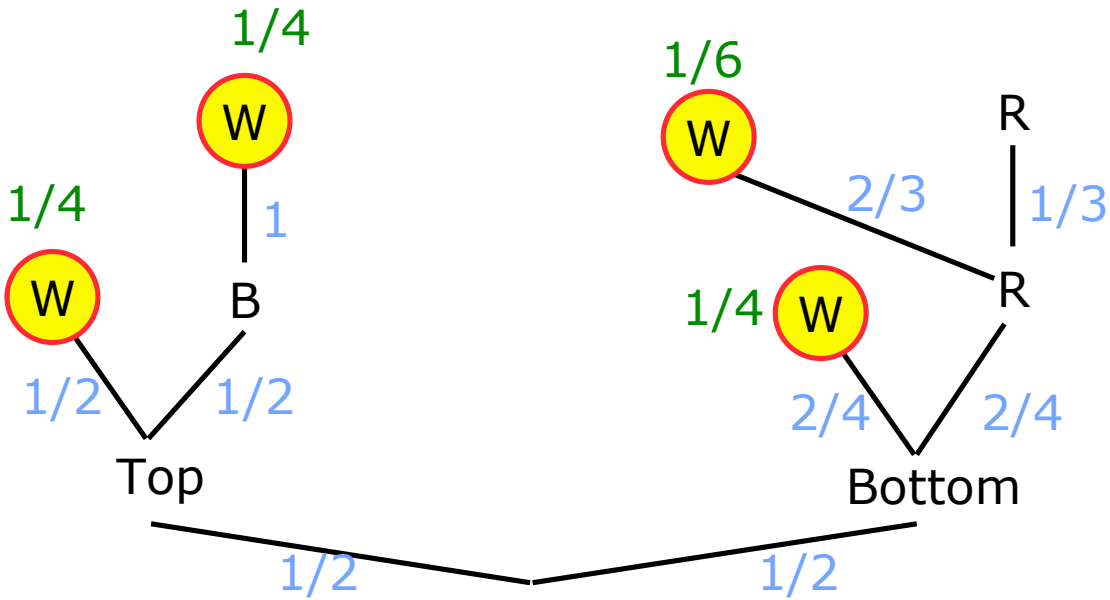
(b) What is the probability he pulls out exactly 2 t-shirts?

(c) What is the probability he is taking t-shirts out of the top drawer if you know that he takes out exactly 2 t-shirts?

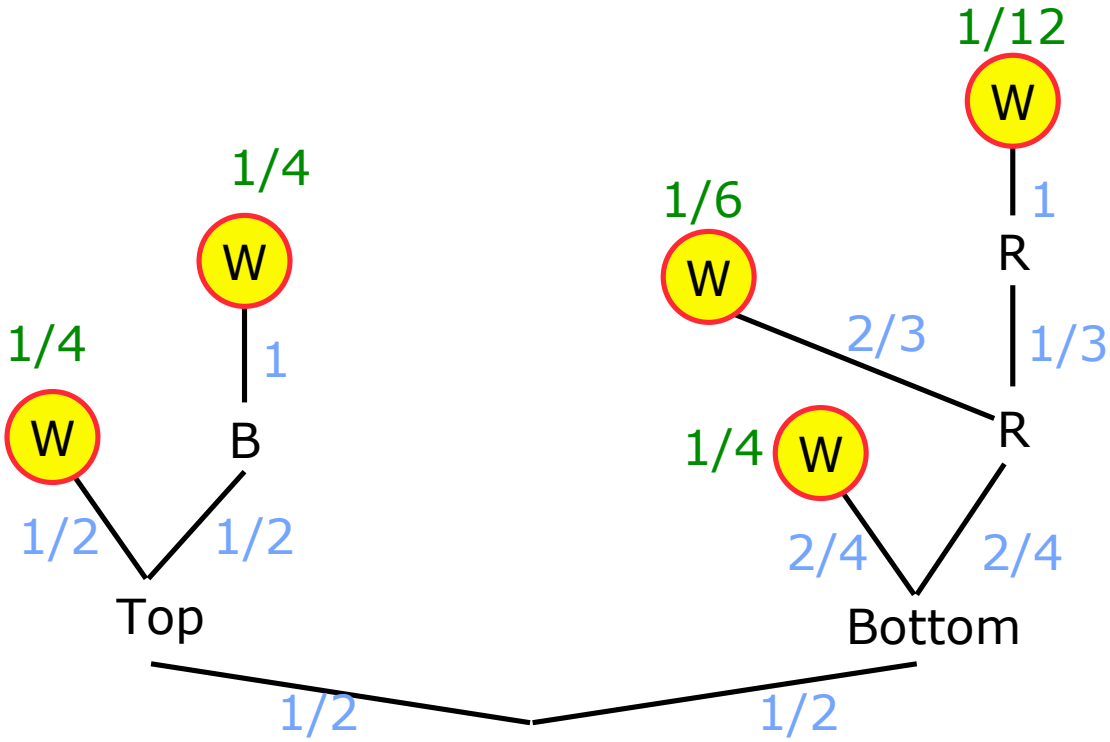
Top — 1 white and 1 blue
Bottom — 2 white and 2 red



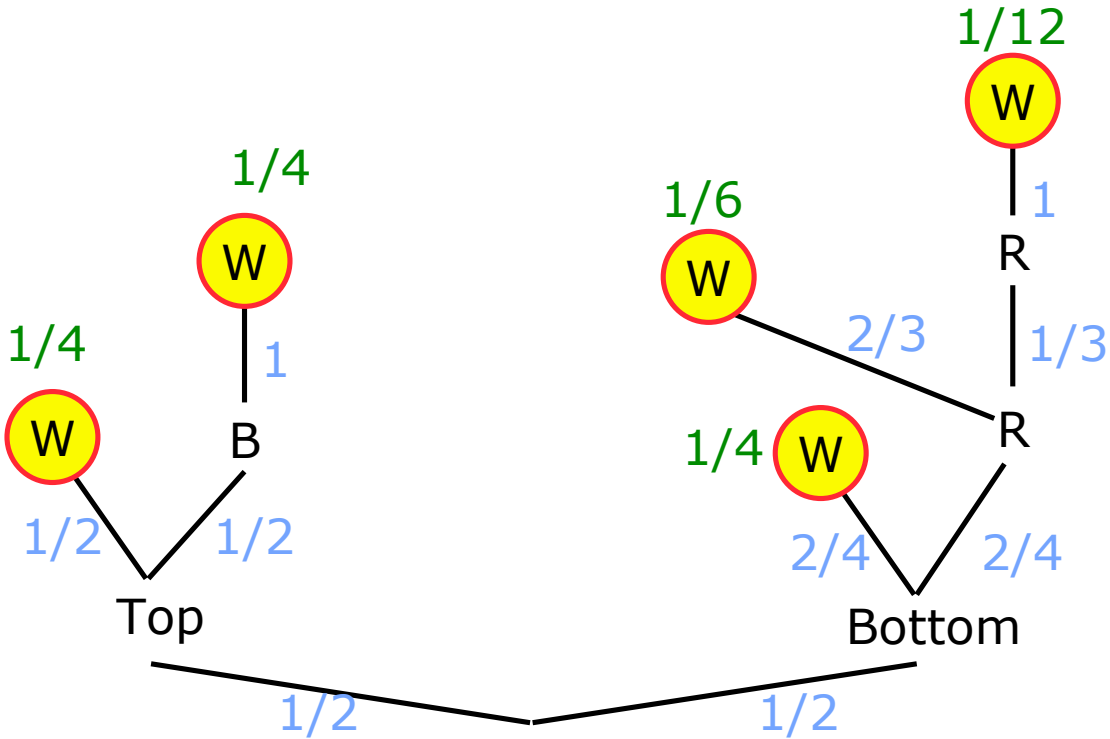
Top — 1 white and 1 blue
Bottom — 2 white and 2 red



Top — 1 white and 1 blue
Bottom — 2 white and 2 red

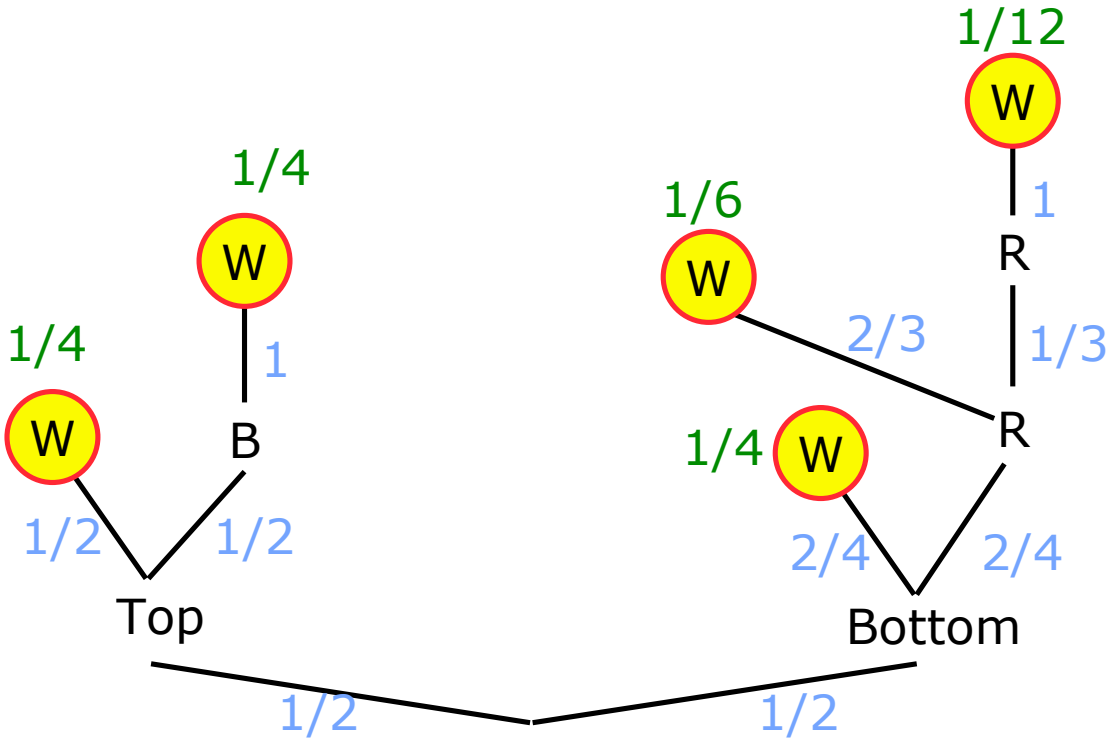


What is the probability he pulls out exactly 2 t-shirts?

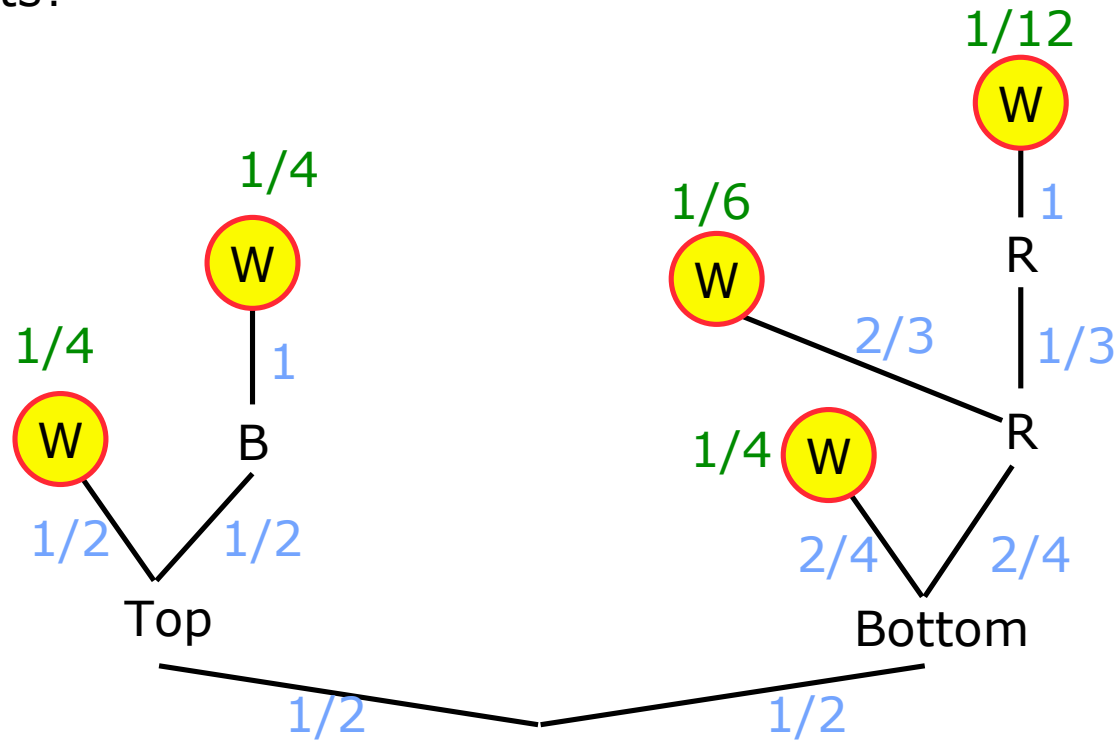


What is the probability he pulls out exactly 2 t-shirts?

Answer: 1/4 + 1/6 = 5/12

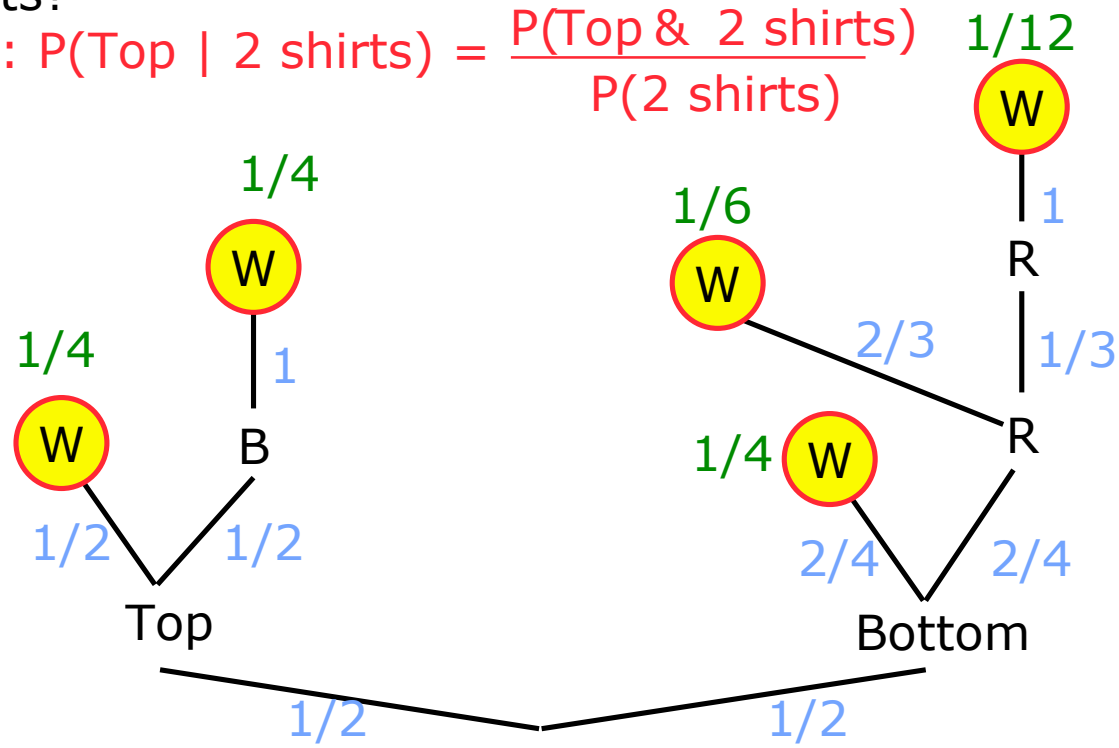


What is the probability he is taking t-shirts out of the top drawer if you know that he takes out exactly 2 t-shirts?



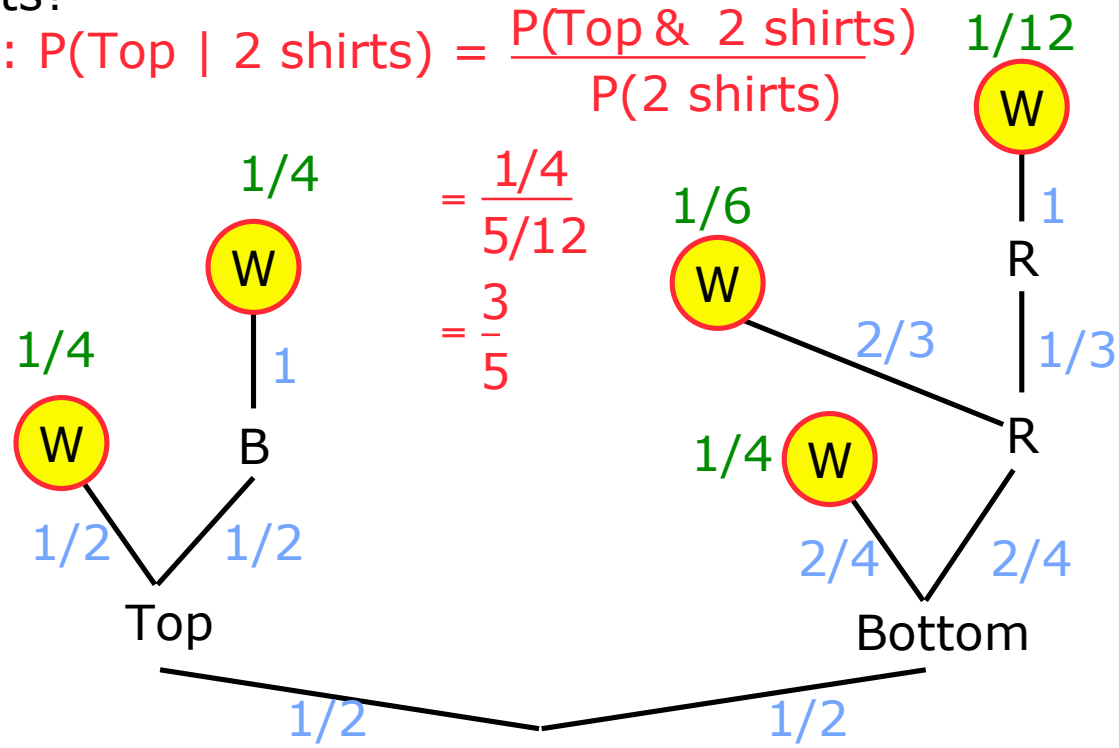
What is the probability he is taking t-shirts out of the top drawer if you know that he takes out exactly 2 t-shirts?

Answer: $P(\text{Top} \mid 2 \text{ shirts}) = \dfrac{P(\text{Top \& 2 shirts})}{P(2 \text{ shirts})}$



What is the probability he is taking t-shirts out of the top drawer if you know that he takes out exactly 2 t-shirts?

Answer: $P(\text{Top} \mid 2 \text{ shirts}) = \dfrac{P(\text{Top \& 2 shirts})}{P(2 \text{ shirts})}$



$= \dfrac{1/4}{5/12}$

$= \dfrac{3}{5}$

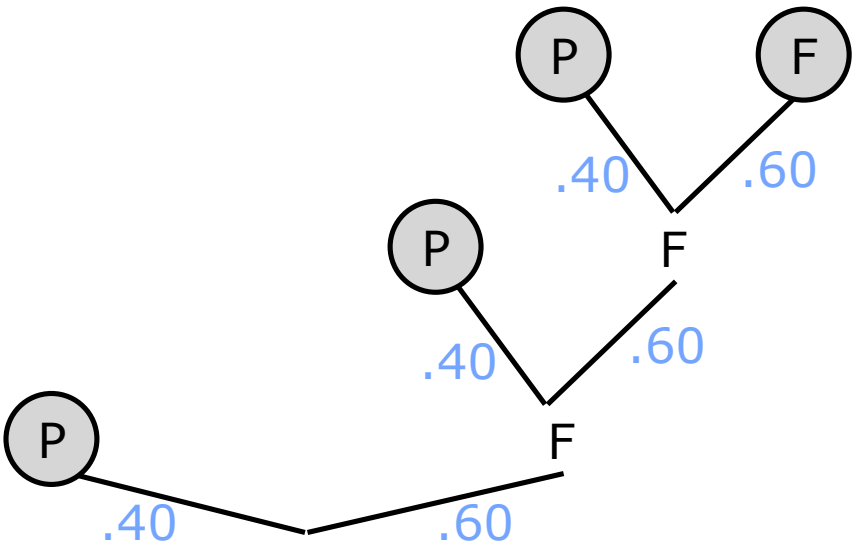
Passing a Qualifying Exam

Tanya wants to pass a qualifying exam, and she is allowed 3 attempts at passing the exam if necessary. Each time she takes the exam there is a 40% chance she will pass.

(a) Draw a tree diagram for this process. [Remember, if she passes the exam she doesn't need to take it again.]

(b) What is the probability she will pass the exam?

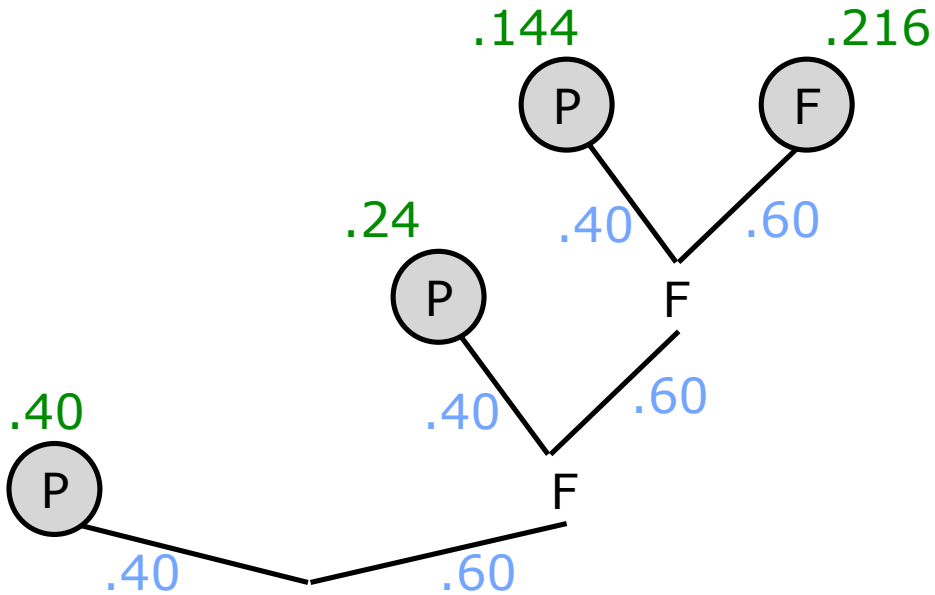
(c) If she fails the first attempt, what then is the conditional probability she will pass?



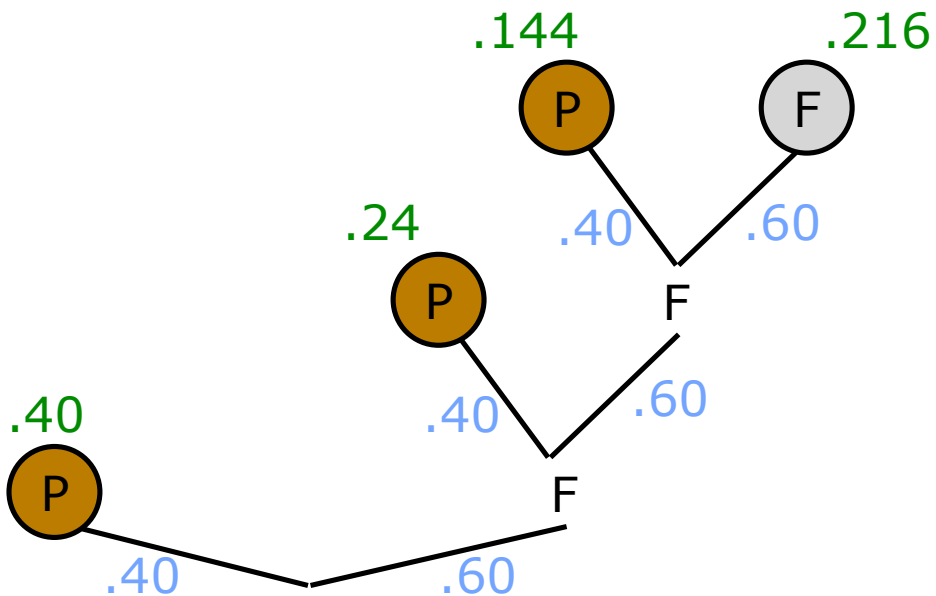
$.60 \times .40 = .24$

$.60 \times .60 \times .40 = .114$

$.60 \times .60 \times .60 = .216$

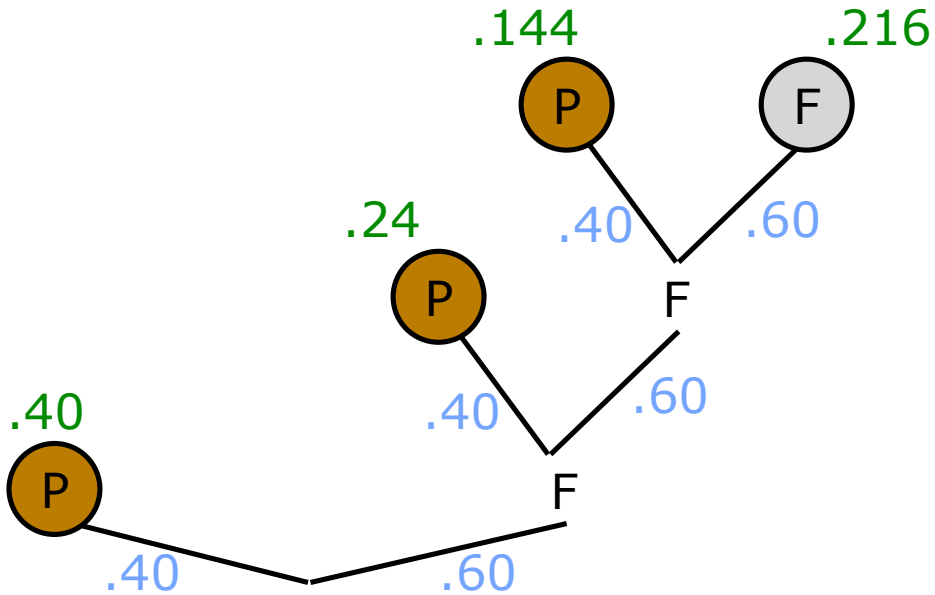


What is the probability she will pass the exam?

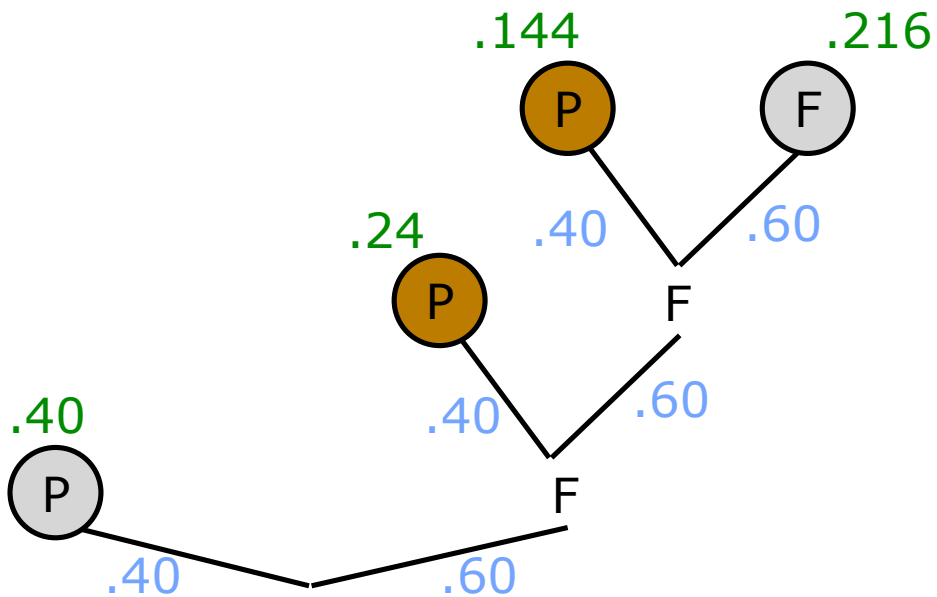


What is the probability she will pass the exam?

Answer: .40 + .24 + .144 = .784



If she fails the first attempt, what then is the conditional probability she will pass?



If she fails the first attempt, what then is the conditional probability she will pass?

P(passes | fails 1st attempt)

$$= \frac{P(\text{passes \& fails 1st attempt})}{P(\text{fails 1st attempt})}$$

$$= \frac{.24 + .144}{.60} = .64$$