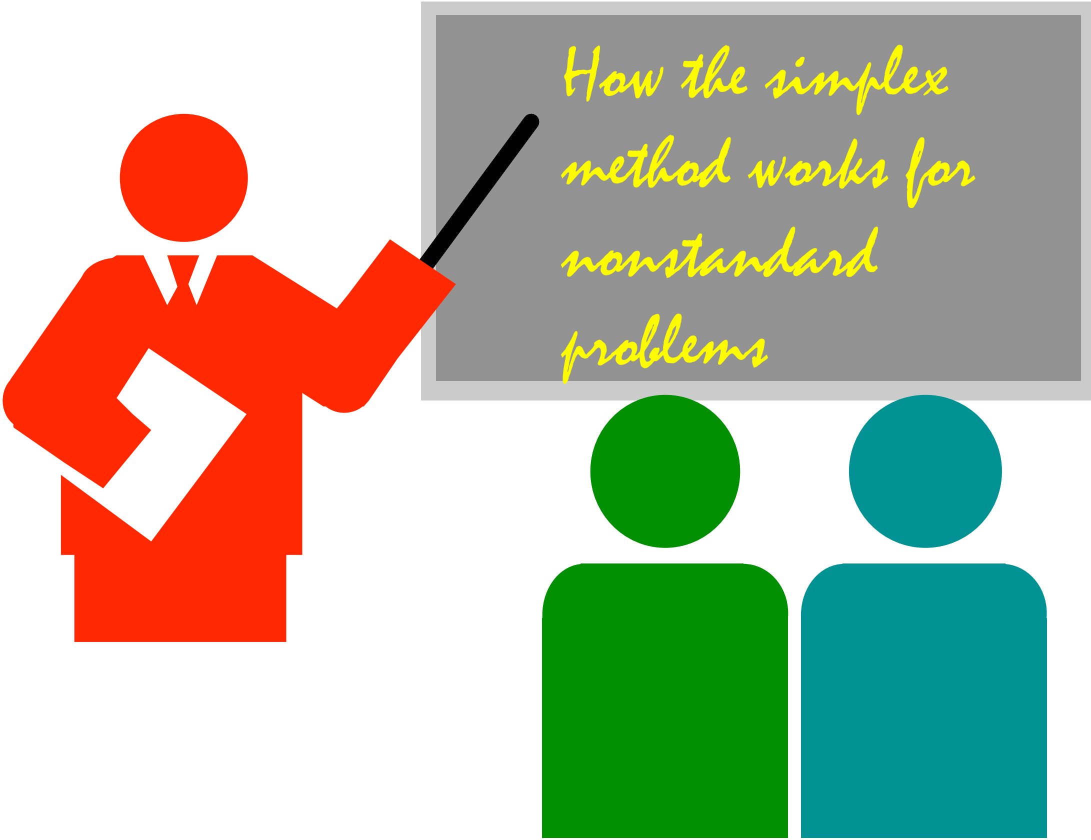

How the simplex method works for nonstandard problems

Example - bicycle factories

A small business makes 3-speed and 10-speed bicycles at two different factories. Factory A produces 16 3-speed and 20 10-speed bikes in one day while factory B produces 12 3-speed and 20 10-speed bikes daily. It costs $1000/day to operate factory A and $800/day to operate factory B. An order for 96 3-speed bikes and 140 10-speed bikes has just arrived. How many days should each factory be operated in order to fill this order at a minimum cost? What is the minimum cost?

x = # days factory A is operated
y = # days factory B is operated

Bicycles

x = # days factory A operated
y = # days factory B operated

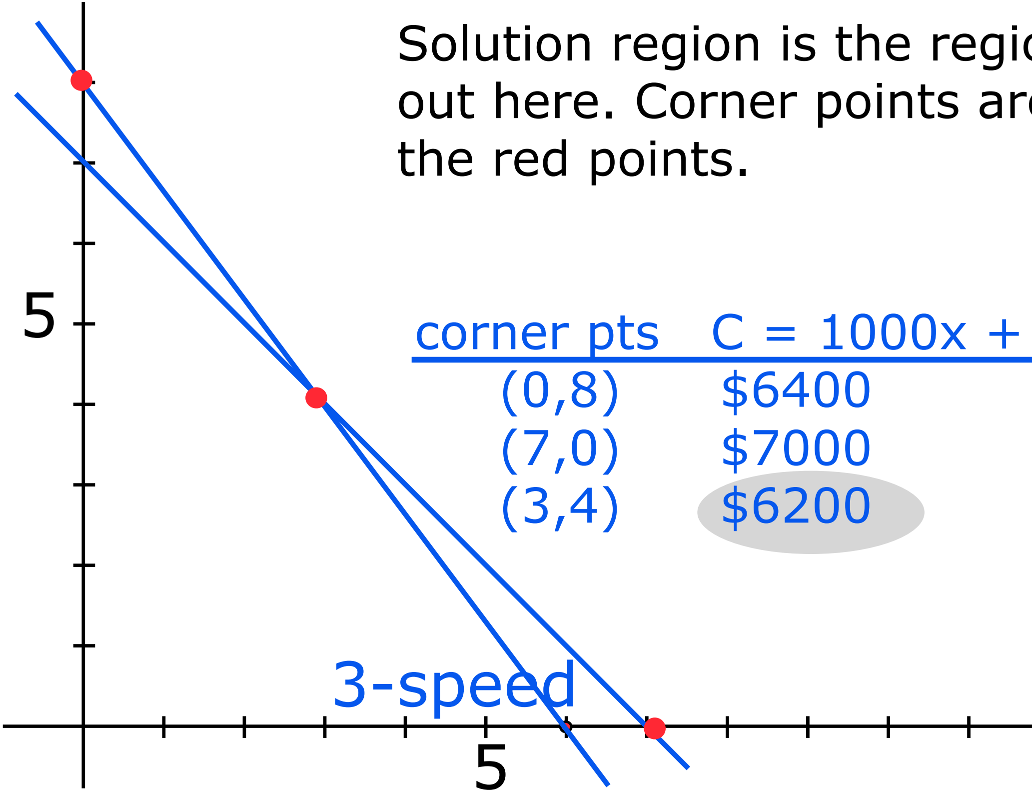
3-speed : $16x + 12y \geq 96$
10-speed : $20x + 20y \geq 140$
$x \geq 0, y \geq 0$

Minimize: $C = 1000x + 800y$

Bicycles

Solution region is the region out here. Corner points are the red points.

corner pts	C = 1000x + 800y
(0,8)	\$6400
(7,0)	\$7000
(3,4)	\$6200

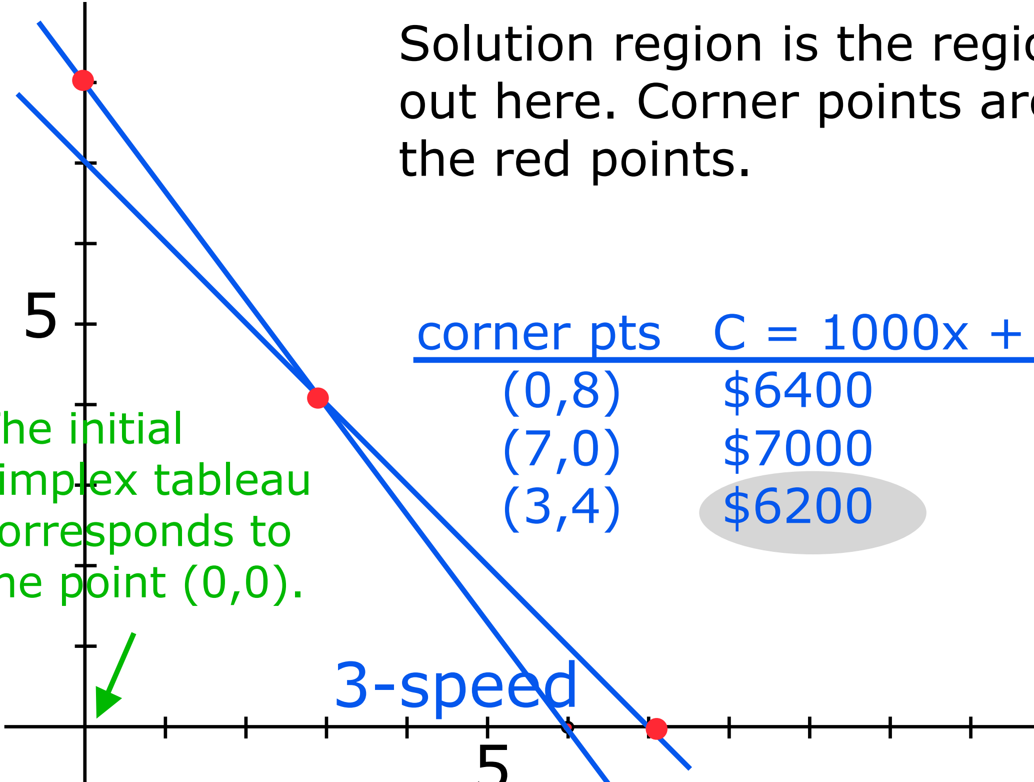


Initial Tableau

$$\begin{array}{ccccc|c} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\ -16 & -12 & 1 & 0 & 0 & -96 \\ -20 & -20 & 0 & 1 & 0 & -140 \\ 1000 & 800 & 0 & 0 & 1 & 0 \end{array}$$

Here the x and y columns are not in unit form, so this simplex tableau represents the point x=0, y=0.

Bicycles



Solution region is the region out here. Corner points are the red points.

corner pts	C = 1000x + 800y
(0,8)	$6400
(7,0)	$7000
(3,4)	$6200

The initial simplex tableau corresponds to the point (0,0).

$$\begin{array}{ccccc} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} \end{array}$$

$$\left[\begin{array}{ccccc|c} -16 & -12 & 1 & 0 & 0 & -96 \\ -20 & -20 & 0 & 1 & 0 & -140 \\ 1000 & 800 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\frac{-96}{-12} = 8 \qquad \frac{-140}{-20} = 7$$

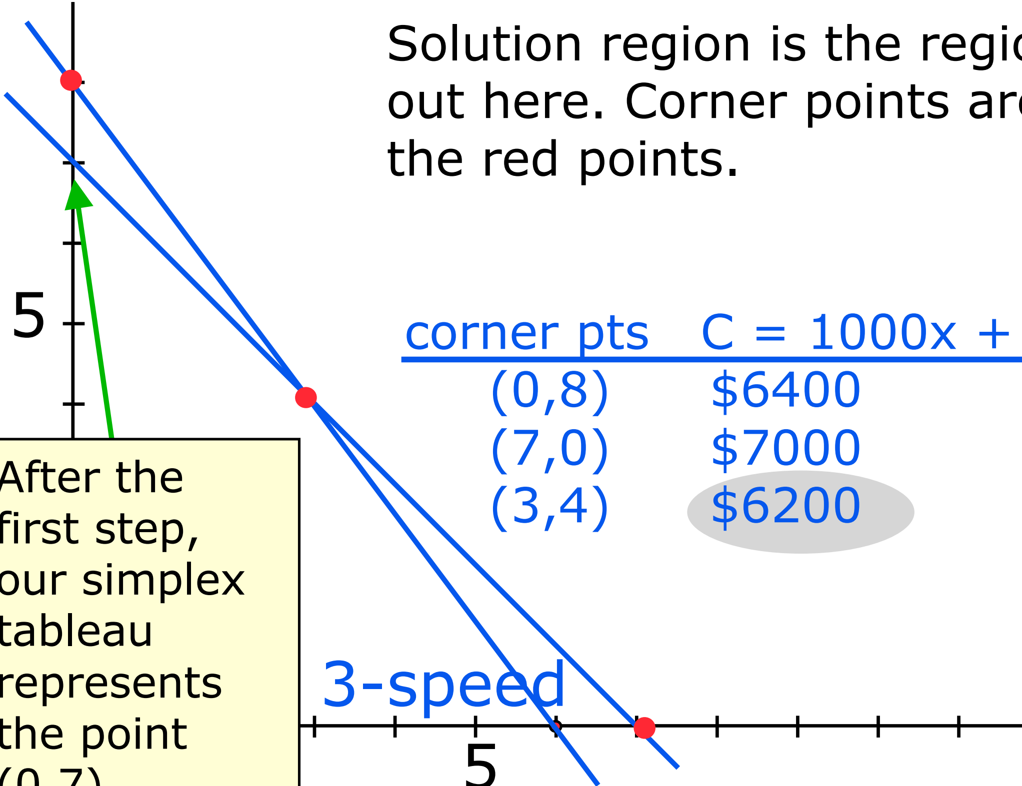
If the 2nd column is used as the pivot column, then -20 must be the pivot element.

$$
\begin{array}{ccccc|c}
\underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\
-4 & 0 & 1 & \frac{-3}{5} & 0 & \boxed{-12} \\
1 & 1 & 0 & \frac{-1}{20} & 0 & 7 \\
200 & 0 & 0 & 40 & 1 & -5600
\end{array}
$$

This tableau corresponds to the point x=0, y=7. Notice that there is still a negative number in the far right column above the bottom entry.

Bicycles

Solution region is the region out here. Corner points are the red points.



corner pts	C = 1000x + 800y
(0,8)	$6400
(7,0)	$7000
(3,4)	$6200

After the first step, our simplex tableau represents the point (0,7)

$$\begin{array}{ccccc|c} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\ -4 & 0 & 1 & \frac{-3}{5} & 0 & -12 \\ 1 & 1 & 0 & \frac{-1}{20} & 0 & 7 \\ 200 & 0 & 0 & 40 & 1 & -5600 \end{array}$$

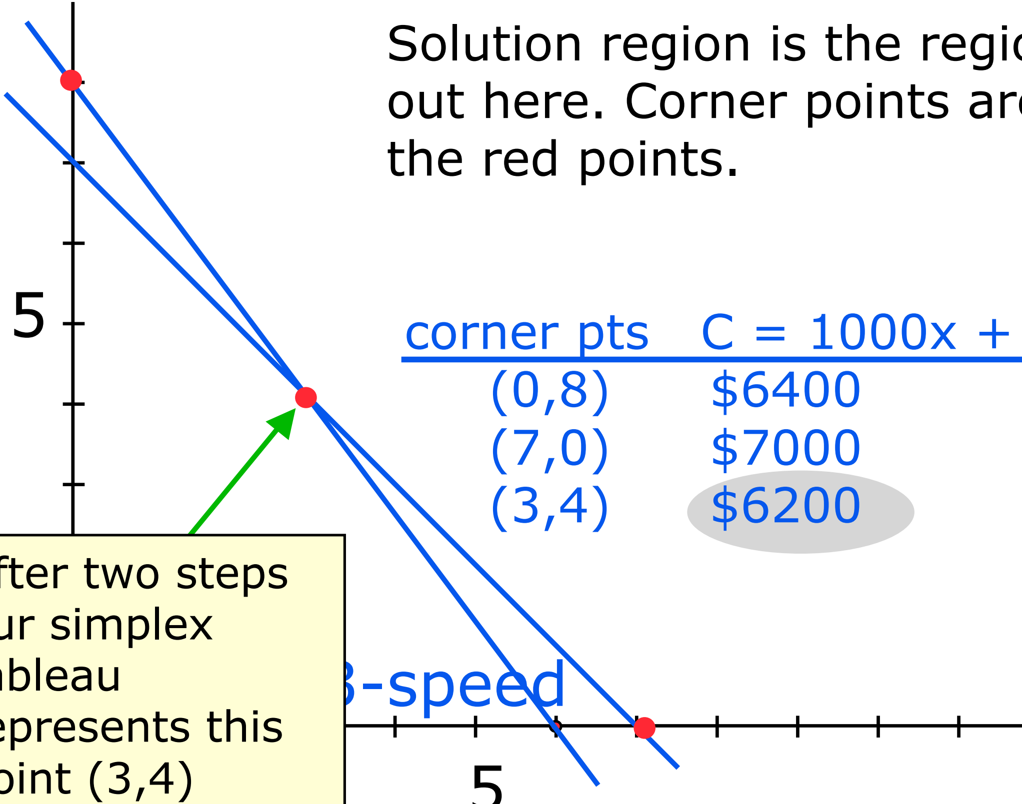
After the first step, this is what our simplex tableau looks like. The fact that there's a negative number on the right is a signal that we're not yet in the solution region.

$$
\begin{array}{ccccc|c}
\underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\
1 & 0 & \frac{-1}{4} & \frac{3}{20} & 0 & 3 \\
0 & 1 & \frac{1}{4} & \frac{-1}{5} & 0 & 4 \\
0 & 0 & 50 & 10 & 1 & -6200
\end{array}
$$

After next step this is what the tableau looks like. Notice that now there is no negative on the right, and furthermore there's no negative in the bottom row. (Bottom right corner doesn't matter.) Optimal solution is:

x=3, y=4, D = -6200, so minimum value of C = +6200.

Bicycles



Solution region is the region out here. Corner points are the red points.

corner pts	C = 1000x + 800y
(0,8)	\$6400
(7,0)	\$7000
(3,4)	\$6200

After two steps our simplex tableau represents this point (3,4)

$$\begin{array}{ccccc|c} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\ 1 & 0 & \frac{-1}{4} & \frac{3}{20} & 0 & 3 \\ 0 & 1 & \frac{1}{4} & \frac{-1}{5} & 0 & 4 \\ 0 & 0 & 50 & 10 & 1 & -6200 \end{array}$$

At this point, we've finished. There are no more negatives on the far right, and none in the bottom row. Optimal solution is:
x=3, y=4, D = -6200, so minimum value of C = +6200.

Now let's backtrack to where we were after the first step and see what would have happened if we had chosen column 4 as our pivot column instead of column 1.

$$
\begin{array}{ccccc|c}
\underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\
-4 & 0 & 1 & \frac{-3}{5} & 0 & -12 \\
1 & 1 & 0 & \frac{-1}{20} & 0 & 7 \\
200 & 0 & 0 & 40 & 1 & -5600
\end{array}
$$

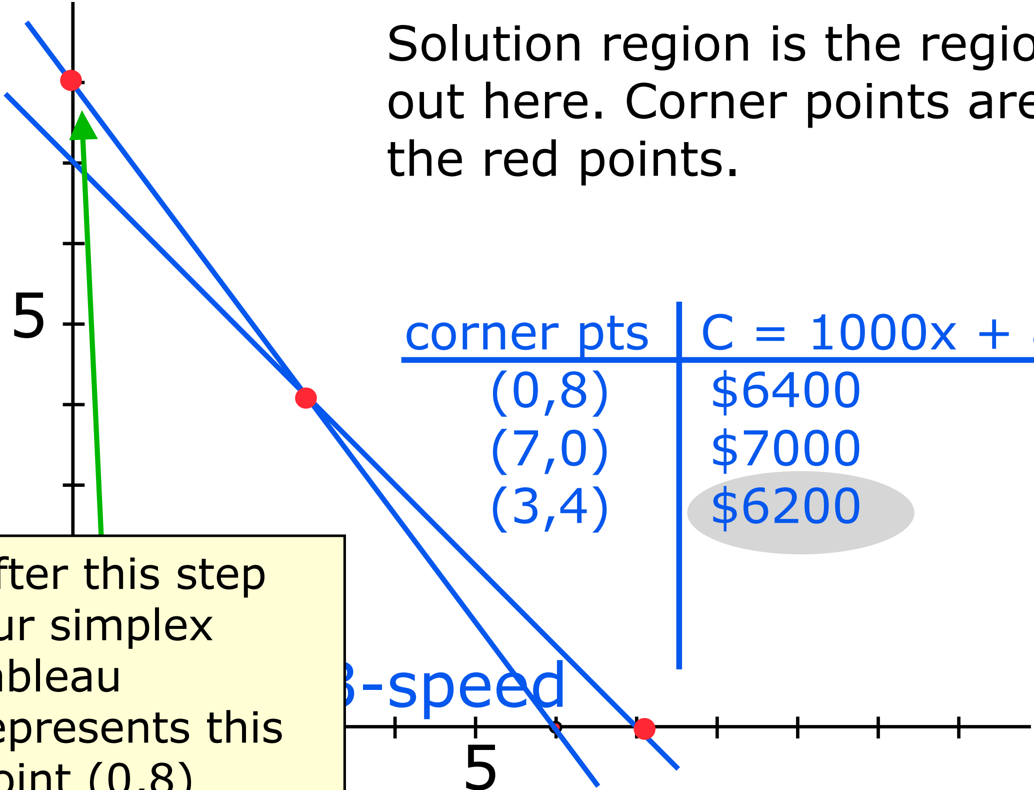
Now let's backtrack to where we were after the first step and see what would have happened if we had chosen column 4 as our pivot column instead of column 1.

$$
\begin{array}{ccccc|c}
\underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\
-4 & 0 & 1 & \frac{-3}{5} & 0 & -12 \\
1 & 1 & 0 & \frac{-1}{20} & 0 & 7 \\
200 & 0 & 0 & 40 & 1 & -5600
\end{array}
$$

$$
\begin{array}{ccccc|c}
\underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} & \\
\frac{20}{3} & 0 & \frac{-5}{3} & 1 & 0 & 20 \\
\frac{4}{3} & 1 & \frac{-1}{12} & 0 & 0 & 8 \\
\frac{-200}{3} & 0 & \frac{200}{3} & 0 & 1 & -6400
\end{array}
$$

We're at the point x=0, y=8. (Make sure you see this in the above tableau.) No negatives now on the right but there is a negative in the bottom row. We're in the solution region but not at the best corner point.

Bicycles



Solution region is the region out here. Corner points are the red points.

corner pts	C = 1000x + 800y
(0,8)	$6400
(7,0)	$7000
(3,4)	$6200

After this step our simplex tableau represents this point (0,8)

$$\begin{array}{ccccc} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} \end{array}$$

$$\left[\begin{array}{ccccc|c} \frac{20}{3} & 0 & \frac{-5}{3} & 1 & 0 & 20 \\ \frac{4}{3} & 1 & \frac{-1}{12} & 0 & 0 & 8 \\ \frac{-200}{3} & 0 & \frac{200}{3} & 0 & 1 & -6400 \end{array}\right]$$

Next pivot element.

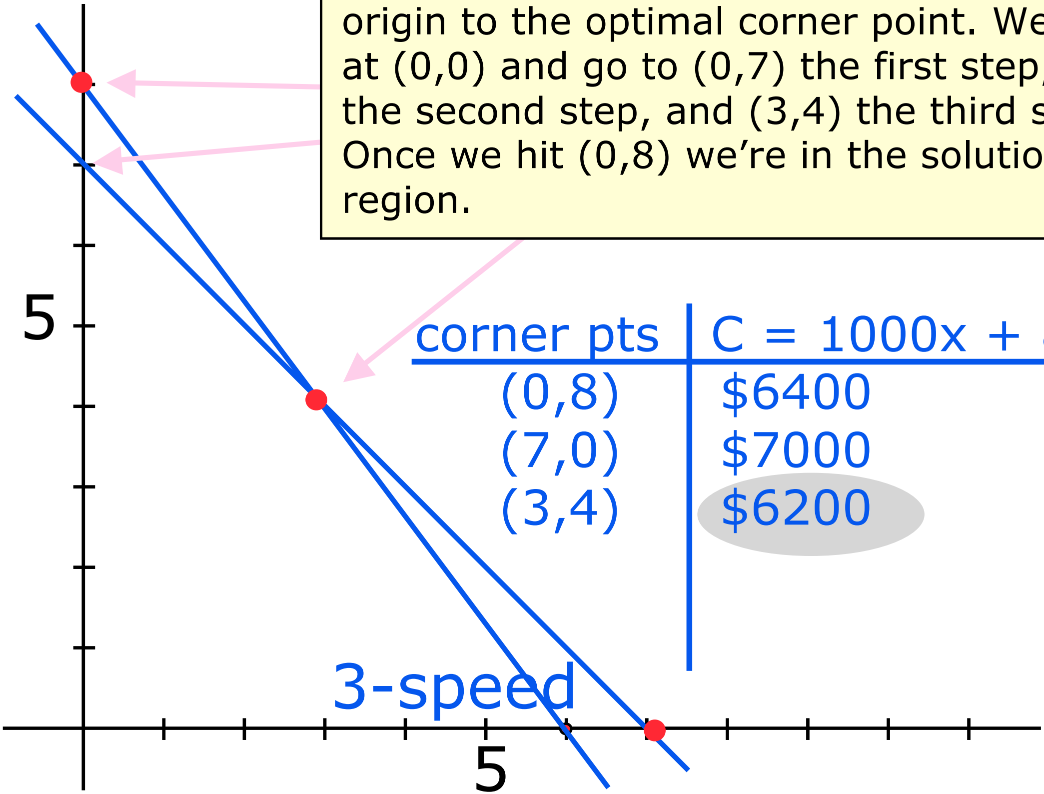
$$\begin{array}{ccccc} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{D} \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & \frac{-1}{4} & \frac{3}{20} & 0 & 3 \\ 0 & 1 & \frac{1}{4} & \frac{-1}{5} & 0 & 4 \\ 0 & 0 & 50 & 10 & 1 & -6200 \end{array}\right]$$

x = 3, y = 4, maximum of D = -6200

miminum value for C = $6200

The method which we just backtracked through takes us 3 steps to go from the origin to the optimal corner point. We start at (0,0) and go to (0,7) the first step, (0,8) the second step, and (3,4) the third step. Once we hit (0,8) we're in the solution region.



corner pts	C = 1000x + 800y
(0,8)	\$6400
(7,0)	\$7000
(3,4)	\$6200