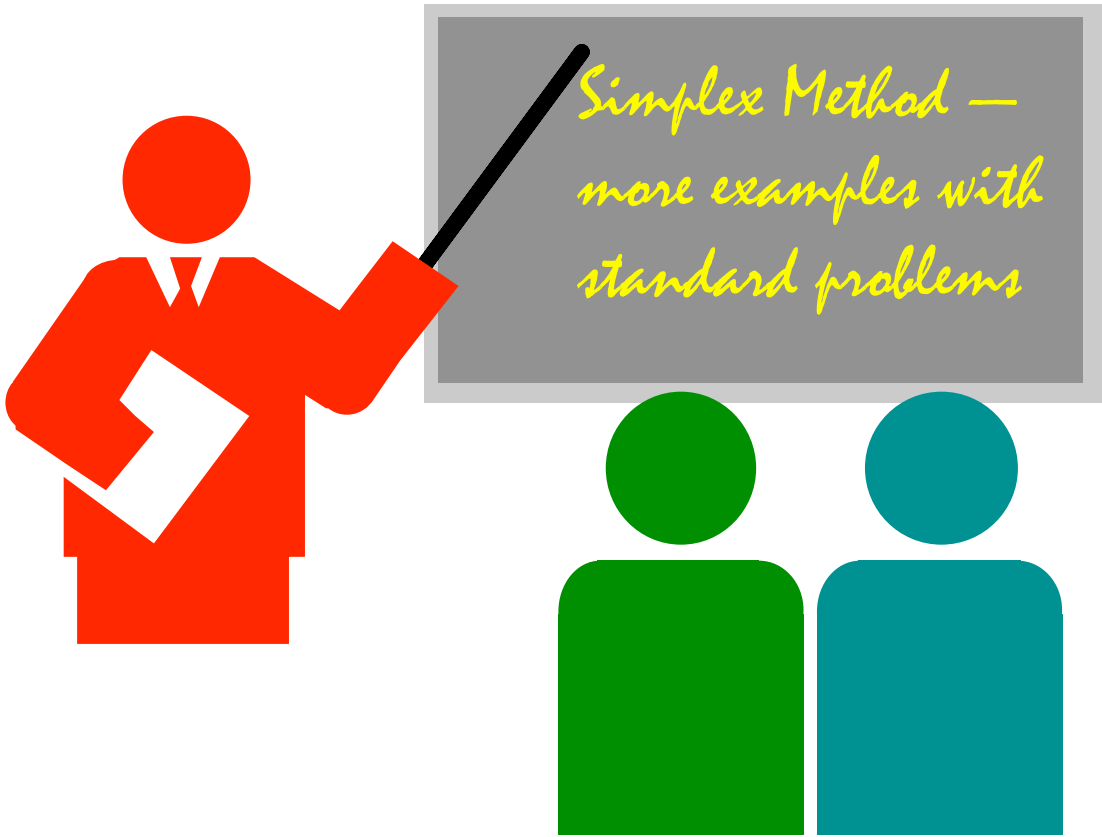

Farmer problem (once again)

A farmer has a 320 acre farm on which she plants two crops: corn and soybeans. For each acre of corn planted, her expenses are $50 and for each acre of soybeans planted, her expenses are $100. Each acre of corn requires 100 bushels of storage and yields a profit of $60; each acre of soybeans requires 40 bushels of storage and yields a profit of $90. If the total amount of storage space available is 19,200 bushels and the farmer has only $20,000 on hand, how many acres of each crop should she plant in order to maximize her profit? What will her profit be if she follows this strategy?

Farmer problem

x	y	u	v	w	P	
1	1	1	0	0	0	320
50	100	0	1	0	0	20000
100	40	0	0	1	0	19200
-60	-90	0	0	0	1	0

Pick the first pivot element.

Farmer problem

x	y	u	v	w	P	
1	1	1	0	0	0	320
50	100	0	1	0	0	20000
100	40	0	0	1	0	19200
-60	-90	0	0	0	1	0

Here's the first pivot element. Now determine the first row operation.

$$\begin{array}{cccccc|c} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \\ \left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 320 \\ \frac{1}{2} & 1 & 0 & \frac{1}{100} & 0 & 0 & 200 \\ 100 & 40 & 0 & 0 & 1 & 0 & 19200 \\ -60 & -90 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

The pivot element has been made into 1.
What are the next row operations?

$$
\begin{array}{cccccc|c}
\underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \\
\left[\begin{array}{cccccc|c}
\frac{1}{2} & 0 & 1 & \frac{-1}{100} & 0 & 0 & 120 \\
\frac{1}{2} & 1 & 0 & \frac{1}{100} & 0 & 0 & 200 \\
80 & 0 & 0 & \frac{-2}{5} & 1 & 0 & 11200 \\
-15 & 0 & 0 & \frac{9}{10} & 0 & 1 & 18000
\end{array}\right]
\end{array}
$$

This completes the first step. Now give an interpretation of this tableau.

$$\begin{array}{cccccc|c} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \\ \left[\begin{array}{cccccc|c} \frac{1}{2} & 0 & 1 & \frac{-1}{100} & 0 & 0 & 120 \\ \frac{1}{2} & 1 & 0 & \frac{1}{100} & 0 & 0 & 200 \\ 80 & 0 & 0 & \frac{-2}{5} & 1 & 0 & 11200 \\ -15 & 0 & 0 & \frac{9}{10} & 0 & 1 & 18000 \end{array}\right] \end{array}$$

This completes the first step: $x = 0$, $v = 0$, $u = 120$, $y = 200$, $w = 11200$, $P = 18000$

Find the next pivot element.

x	y	u	v	w	P	
$\frac{1}{2}$	0	1	$\frac{-1}{100}$	0	0	120
$\frac{1}{2}$	1	0	$\frac{1}{100}$	0	0	200
80	0	0	$\frac{-2}{5}$	1	0	11200
-15	0	0	$\frac{9}{10}$	0	1	18000

Here's the next pivot element.
What is the next row operation?

$$\begin{array}{c} \begin{matrix} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \end{matrix} \\ \left[\begin{array}{cccccc|c} \frac{1}{2} & 0 & 1 & \frac{-1}{100} & 0 & 0 & 120 \\ \frac{1}{2} & 1 & 0 & \frac{1}{100} & 0 & 0 & 200 \\ 1 & 0 & 0 & \frac{-1}{200} & \frac{1}{80} & 0 & 140 \\ -15 & 0 & 0 & \frac{9}{10} & 0 & 1 & 18000 \end{array}\right] \end{array}$$

The pivot element has been made into 1. Specify the remaining row operations in this step.

$$\begin{array}{cccccc|c} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \\ 0 & 0 & 1 & \frac{-3}{400} & \frac{-1}{160} & 0 & 50 \\ 0 & 1 & 0 & \frac{1}{80} & \frac{-1}{160} & 0 & 130 \\ 1 & 0 & 0 & \frac{-1}{200} & \frac{1}{80} & 0 & 140 \\ 0 & 0 & 0 & \frac{33}{40} & \frac{3}{16} & 1 & 20100 \end{array}$$

After the second step this is what the tableau looks like. Interpret it now.

$$\begin{array}{cccccc|c} \underline{x} & \underline{y} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \\ 0 & 0 & 1 & \frac{-3}{400} & \frac{-1}{160} & 0 & 50 \\ 0 & 1 & 0 & \frac{1}{80} & \frac{-1}{160} & 0 & 130 \\ 1 & 0 & 0 & \frac{-1}{200} & \frac{1}{80} & 0 & 140 \\ 0 & 0 & 0 & \frac{33}{40} & \frac{3}{16} & 1 & 20100 \end{array}$$

After the second step: v=0, w=0, u=50, y=130, x=140, P=$20,100. This is the optimal solution since there are no negative numbers in the bottom row.

Planning Subdivision

A developer is planning to build a new subdivision consisting of townhouses, single-story detached houses and two-story detached houses. On one acre he can put 6 townhouses or 4 single-story houses or 2 two-story houses and he has 60 acres available. It costs him $40,000 to build each townhouse, $50,000 to build each single-story house and $60,000 to build each two-story house. He makes a profit of $15,000 on each townhouse, $18,000 on each single-story house and $20,000 on each two-story house and he has $2,880,000 of capital available. Townhouses require 2500 hours of labor, single-story houses require 3000 hours of labor and two-story houses require 4000 hours of labor and he has 240,000 hours of labor available. How many houses of each type should he construct in order to maximize his profit?

Planning Subdivision

x = # townhouses
y = # single story
z = # 2-story

Planning Subdivision

x = # townhouses
y = # single story
z = # 2-story

$1/6\ x + 1/4\ y + 1/2\ z \leq 60$
$40x + 50y + 60z \leq 2880$
$25x + 30y + 40z \leq 2400$

Maximize:
$P = 15x + 18y + 20z$ (in thousands of $)

$$1/6\ x + 1/4\ y + 1/2\ z \leq 60$$
$$40x + 50y + 60z \leq 2880$$
$$25x + 30y + 40z \leq 2400$$

$$1/6\ x + 1/4\ y + 1/2\ z + u = 60$$
$$40x + 50y + 60z + v = 2880$$
$$25x + 30y + 40z + w = 2400$$
$$-15x - 18y - 20z + P = 0$$

$$\begin{array}{cccccccc} \underline{x} & \underline{y} & \underline{z} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \\ \left[\begin{array}{ccccccc} \frac{1}{6} & \frac{1}{4} & \frac{1}{2} & 1 & 0 & 0 & 0 & 60 \\ 40 & 50 & 60 & 0 & 1 & 0 & 0 & 2880 \\ 25 & 30 & 40 & 0 & 0 & 1 & 0 & 2400 \\ -15 & -18 & -20 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

Find the first pivot element.

x	y	z	u	v	w	P	
$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	60
40	50	60	0	1	0	0	2880
25	30	40	0	0	1	0	2400
-15	-18	-20	0	0	0	1	0

Here is the first pivot element.
What is the first row operation?

$$
\begin{array}{cccccccc}
\underline{x} & \underline{y} & \underline{z} & \underline{u} & \underline{v} & \underline{w} & \underline{P} & \\
\left[\begin{array}{ccccccc}
\frac{1}{6} & \frac{1}{4} & \frac{1}{2} & 1 & 0 & 0 & 0 \\
\frac{2}{3} & \frac{5}{6} & 1 & 0 & \frac{1}{60} & 0 & 0 \\
25 & 30 & 40 & 0 & 0 & 1 & 0 \\
-15 & -18 & -20 & 0 & 0 & 0 & 1
\end{array}\right.
&
\left.\begin{array}{c}
60 \\ 48 \\ 2400 \\ 0
\end{array}\right]
\end{array}
$$

Specify the row operations to finish this step.

x	y	z	u	v	w	P	
$\frac{-1}{6}$	$\frac{-1}{6}$	0	1	$\frac{-1}{120}$	0	0	36
$\frac{2}{3}$	$\frac{5}{6}$	1	0	$\frac{1}{60}$	0	0	48
$\frac{-5}{3}$	$\frac{-10}{3}$	0	0	$\frac{-2}{3}$	1	0	480
$\frac{-5}{3}$	$\frac{-4}{3}$	0	0	$\frac{1}{3}$	0	1	960

After the above step, here's our tableau. Interpret it now.

x	y	z	u	v	w	P	
$\frac{-1}{6}$	$\frac{-1}{6}$	0	1	$\frac{-1}{120}$	0	0	36
$\frac{2}{3}$	$\frac{5}{6}$	1	0	$\frac{1}{60}$	0	0	48
$\frac{-5}{3}$	$\frac{-10}{3}$	0	0	$\frac{-2}{3}$	1	0	480
$\frac{-5}{3}$	$\frac{-4}{3}$	0	0	$\frac{1}{3}$	0	1	960

After the above step: x=0, y=0, v=0, u=36, z=48, w=480, P=960.
Find the next pivot element.

x	y	z	u	v	w	P	
$\frac{-1}{6}$	$\frac{-1}{6}$	0	1	$\frac{-1}{120}$	0	0	36
$\frac{2}{3}$	$\frac{5}{6}$	1	0	$\frac{1}{60}$	0	0	48
$\frac{-5}{3}$	$\frac{-10}{3}$	0	0	$\frac{-2}{3}$	1	0	480
$\frac{-5}{3}$	$\frac{-4}{3}$	0	0	$\frac{1}{3}$	0	1	960

Here is the pivot element.
Make it into a 1.

x	y	z	u	v	w	P	
$\frac{-1}{6}$	$\frac{-1}{6}$	0	1	$\frac{-1}{120}$	0	0	36
1	$\frac{5}{4}$	$\frac{3}{2}$	0	$\frac{1}{40}$	0	0	72
$\frac{-5}{3}$	$\frac{-10}{3}$	0	0	$\frac{-2}{3}$	1	0	480
$\frac{-5}{3}$	$\frac{-4}{3}$	0	0	$\frac{1}{3}$	0	1	960

What are the remaining row operations for this step?

x	y	z	u	v	w	P	
0	$\frac{1}{24}$	$\frac{1}{4}$	1	$\frac{-1}{240}$	0	0	48
1	$\frac{5}{4}$	$\frac{3}{2}$	0	$\frac{1}{40}$	0	0	72
0	$\frac{-5}{4}$	$\frac{5}{2}$	0	$\frac{-5}{8}$	1	0	600
0	$\frac{3}{4}$	$\frac{5}{2}$	0	$\frac{3}{8}$	0	1	1080

The optimal solution has been found since there are no negatives in bottom row. Interpret this final tableau.

x	y	z	u	v	w	P	
0	$\frac{1}{24}$	$\frac{1}{4}$	1	$\frac{-1}{240}$	0	0	48
1	$\frac{5}{4}$	$\frac{3}{2}$	0	$\frac{1}{40}$	0	0	72
0	$\frac{-5}{4}$	$\frac{5}{2}$	0	$\frac{-5}{8}$	1	0	600
0	$\frac{3}{4}$	$\frac{5}{2}$	0	$\frac{3}{8}$	0	1	1080

Optimal solution: y=0, z=0, v=0, u=48, x=72, w=600, P=$1080 thousand = $1,080,000. The developer should build only townhouses.