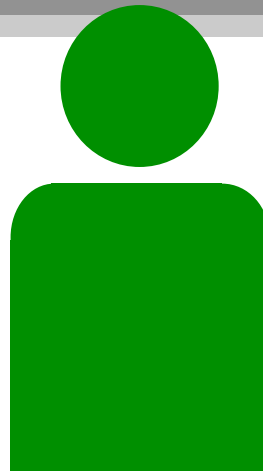
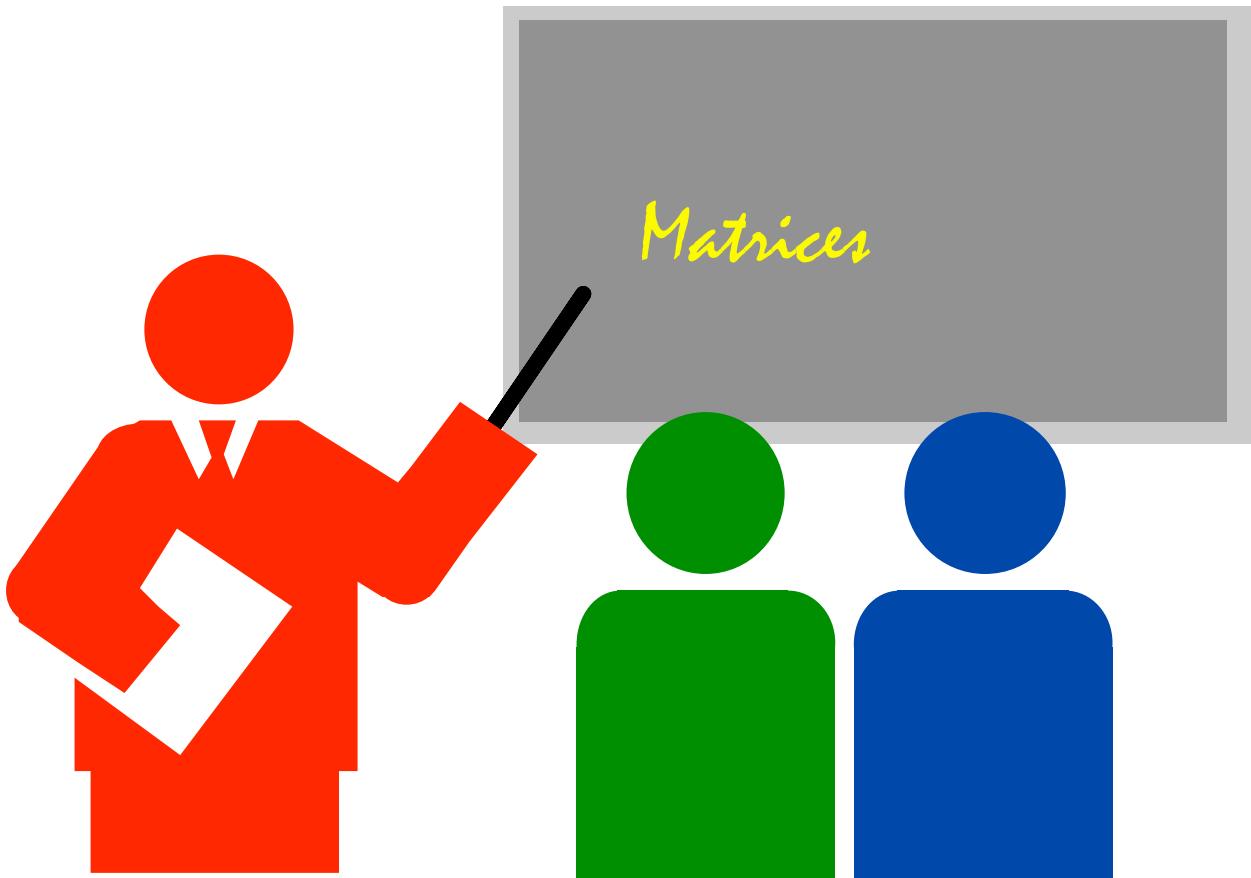


Matrices
linear programming
probability





What is a matrix?

$$A = \begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix}$$

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This matrix has 2 rows and 3 columns and is called a 2×3 matrix.

$$B = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 7 & 4 & 4 & 3 \\ -1 & 2 & 3 & 3 \end{bmatrix}$$

This matrix has 3 rows and 4 columns, so it is a **3 × 4** matrix.

How are matrices added?

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$$\begin{bmatrix} 2 & 3 & 3 \\ -1 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 5 & -4 & 1 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

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The two matrices to be added must be the same size.

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$$\begin{bmatrix} 2 & 3 & 3 \\ -1 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 5 & -4 & 1 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -1 & 4 \\ -1 & 4 & 9 \end{bmatrix}$$

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How are matrices subtracted?

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$$\begin{bmatrix} 2 & 3 & 3 \\ -1 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 5 & -4 & 1 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

How are matrices subtracted?

$$\begin{bmatrix} 2 & 3 & 3 \\ -1 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 5 & -4 & 1 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

To subtract two matrices, the matrices must be the same size.

How are matrices subtracted?

$$\begin{bmatrix} 2 & 3 & 3 \\ -1 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 5 & -4 & 1 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} -3 & & \end{bmatrix}$$

How are matrices subtracted?

$$\begin{bmatrix} 2 & 3 & 3 \\ -1 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 5 & -4 & 1 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 7 & \\ & & \end{bmatrix}$$

How are matrices subtracted?

$$\begin{bmatrix} 2 & 3 & 3 \\ -1 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 5 & -4 & 1 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 7 & 2 \\ -1 & -4 & 5 \end{bmatrix}$$

When are matrices equal?

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$$\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

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$$\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 3 & \frac{12}{6} \\ 1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix}$$

The matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is called
the 2×3 zero matrix.

$$\begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix}$$

The matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is called

the 2×3 zero matrix.

When a zero matrix is added to another matrix, the other matrix is not changed.

Multiplying a number times a matrix

$$3 \begin{bmatrix} 7 & 4 & -1 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 12 & -3 \\ 6 & 6 & 9 \end{bmatrix}$$

$$3 \times 7 = 21, \text{ etc.}$$

Problem

$$A = \begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 2 \\ 7 & 0 & -3 \end{bmatrix}$$

Find $2A - 3B$

Problem

$$A = \begin{bmatrix} 2 & -7 & 4 \\ 4 & 0 & 18 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 2 \\ 7 & 0 & -3 \end{bmatrix}$$

Find $2A - 3B$

$$2A = \begin{bmatrix} 4 & -14 & 8 \\ 8 & 0 & 36 \end{bmatrix}, \quad 3B = \begin{bmatrix} -3 & 6 & 6 \\ 21 & 0 & -9 \end{bmatrix}$$

Solution

$$2A = \begin{bmatrix} 4 & -14 & 8 \\ 8 & 0 & 36 \end{bmatrix}, \quad 3B = \begin{bmatrix} -3 & 6 & 6 \\ 21 & 0 & -9 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 7 & -20 & 2 \\ -13 & 0 & 45 \end{bmatrix}$$

Problem

$$3 \begin{bmatrix} y & 4 \\ 0 & 2z \\ x & 2 \end{bmatrix} = 4 \begin{bmatrix} 1 & 4 \\ u & z \\ 2 & 2 \end{bmatrix} - 2 \begin{bmatrix} -1 & -x \\ -2 & -3 \\ 7 & u \end{bmatrix}$$

Problem

$$3 \begin{bmatrix} \textcircled{y} & 4 \\ 0 & 2z \\ x & 2 \end{bmatrix} = 4 \begin{bmatrix} \textcircled{1} & 4 \\ u & z \\ 2 & 2 \end{bmatrix} - 2 \begin{bmatrix} \textcircled{-1} & -x \\ -2 & -3 \\ 7 & u \end{bmatrix}$$

Solution

$$3y = 4 + 2$$

Problem

$$3 \begin{bmatrix} y & \textcircled{4} \\ 0 & 2z \\ x & 2 \end{bmatrix} = 4 \begin{bmatrix} 1 & \textcircled{4} \\ u & z \\ 2 & 2 \end{bmatrix} - 2 \begin{bmatrix} -1 & \textcircled{-x} \\ -2 & -3 \\ 7 & u \end{bmatrix}$$

Solution

$$3y = 4 + 2$$

$$12 = 16 + 2x$$

Problem

$$3 \begin{bmatrix} y & \textcircled{4} \\ 0 & 2z \\ x & 2 \end{bmatrix} = 4 \begin{bmatrix} 1 & \textcircled{4} \\ u & z \\ 2 & 2 \end{bmatrix} - 2 \begin{bmatrix} -1 & \textcircled{-x} \\ -2 & -3 \\ 7 & u \end{bmatrix}$$

Solution

$$3y = 4 + 2$$

$$12 = 16 + 2x$$

$$y = 2$$

$$x = -2$$

You can continue to process the matrices until you arrive at some contradictory information that indicates that the matrix equation can't possibly be true.