MATHEMATICS

- GEOMETRY AND MATHEMATICAL PHYSICS
- LIE ALGEBRAS, GROUPS AND COMBINATORICS
- DYNAMICAL SYSTEMS
- SYMBOLIC COMPUTATION

Geometry and Mathematical Physics

at NCSU

- RESEARCH FACULTY
  - **Amassa Fauntleroy**, Professor Algebraic and Complex Geometry
  - **Ronald Fulp**, Professor Guage theories; fiber bundle formulation, the path-space formulation, cohomological formulations obtained via BRST theory.
  - **Arkady Kheyfets**, Professor quantum gravity and quantum cosmology, quantization of constrained dynamic systems, relativistic smoothed particle hydrodynamics.
  - **Tom Lada**, Professor algebraic topology, strongly homotopy algebraic structures with applications to physics.
  - **Larry Norris**, Associate Professor symplectic geometry, affine connections, geometric mechanics, classical and quantum gravity, and unified field theories and gauge theories.
  - **Irina Kogan**, Assistant Professor Applications of differential geometry and symmetry methods to differential equations.

- STUDENTS
  - 7 Ph.D. students graduated since 2004 with jobs at Max Planck Institute (Potsdam), Math Institute of the Academy of Sciences of Georgia, University of New Orleans, SAS.
  - 5 Ph.D. students in progress

- OTHER ACTIVITY
  - Close ties to the physics research groups in Los Alamos, the University of Sevilla, Spain, the Free University of Brussels, Czech Academy of Sciences, Georgian Academy of Sciences, UNC,Chapel Hill.
  - Organized several Special Sessions at AMS meetings, talks at math institutes, universities and national labs.

Irina Kogan's current research:
Differential and Algebraic Invariants
Two Ph.D. students are involved in this project.

Invariants are functions that are unchanged under a given group of transformations.

Example: Rigid motions (rotations, translations, reflections) on a plane form a group. The distance *between two points is an* algebraic invariant *and the* curvature *of a plane curve is a* differential invariant*.*

We look for methods to generate invariants, study their properties, and apply them in various areas of mathematics, physics and engineering.

Some Applications:

- Geometric study of differential or algebraic equations: find the symmetry group of an equation and use it to solve or simplify the equation.

  Sophus Lie developed the theory of Lie groups to study this problem!

- Equivalence problems: find (simple) canonical representatives for geometric objects or equations. Find a transformation that brings a given object or a given equation to the canonical form. *This problem often arises in engineering, in particular, in mechanics and computer image recognition.*

Methods come from:

- Differential Geometry: manifolds, vector bundles, Lie groups.
- Algebra: groups, commutative rings, fields, Lie algebras.
- Computational algebra: algorithms and software for symbolic computation.

LIE ALGEBRAS, GROUPS AND COMBINATORICS

- RESEARCH FACULTY
  - Ernest Stitzinger, Professor - Lie algebras and their relations to other non-associate algebras and groups.
  - Mohan Putcha, Professor - Algebraic Groups, Algebraic Monoids and their representations.
  - Kailash C. Misra, Professor - Structure and representations of Kac-Moody Lie algebras, quantum groups and their applications.
  - Naihuan Jing, Professor -Infiinite dimensional Lie algebras, quantum groups, and their relations to algebraic combinatorics.
  - Loek Helminck, Professor - Symmetric spaces including comp.aspects. Representations associated with real, p-adic,complex symmetric spaces.
  - Bojko Bakalov, Assistant Professor - Vertex algebras, integrable systems and conformal field theory.
  - Nathan Reading, Assistant Professor - Algebraic and Geometric combinatorics; relations to Lie theory
  - Patricia Hersh, Assistant Professor - Algebraic and Geometric Combinatorics

- PH.D. STUDENTS
  - 14 since 2002. Jobs at Rutgers, UNC-Wilmington, Wake Forrest, Appalachian State, Shaw, Virginia Commonwealth, Lamar, St. Lawrence, Northrop-Grumann, Gonzaga, Microsoft, IBM, NSA
  - 9 present students
- OTHER ACTIVITY
  - Lie Algebras (MA 720) offered each Spring and usually followed by a more advanced course.
  - Invited talks and organizing international conferences on and off campus.
    - NSF funded international conference on ”Lie Algebras, Vertex Operator Algebras and their applications” NCSU May 17-21, 2005.
    - CBMS conference on ”Algebraic Combinatorics” NCSU June 13-17, 2006.

$$\mathbf{SL_2} = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} | ad - bc = 1\} \quad \supset \quad D_2 = \{\begin{pmatrix} q & 0 \\ 0 & q^{-1} \end{pmatrix} | q \neq 0\}$$

$??\rho : SL_2 \to \mathrm{GL}_N =$ invertible matrices $??$

All such are known, built up from basic (irreducible) ones, $\rho_M, M = 1, 2, \ldots.$

$$\rho_M \begin{pmatrix} q & 0 \\ 0 & q^{-1} \end{pmatrix} = \begin{pmatrix} q^M & 0 & \cdots & 0 & 0 \\ 0 & q^{M-2} & \cdots & 0 & 0 \\ & & \ddots & & \\ 0 & 0 & \cdots & q^{2-M} & 0 \\ 0 & 0 & \cdots & 0 & q^{-M} \end{pmatrix}$$

$$tr(\rho_M \begin{pmatrix} q & 0 \\ 0 & q^{-1} \end{pmatrix}) = q^M + q^{M-1} + \ldots + q^{2-M} + q^{-M} = \frac{q^{M+1} - q^{-1-M}}{q - q^{-1}}$$

Can do the same for

"Groups" of "infinite matrices"

$\Downarrow$ $\Downarrow$

Affine Lie Algebras Representations

$$\sum_i a_i q^i = \prod_{n\geq 1} (1-q^{2n-1})^{-1}$$

$a_i = \#$ of ways of writing i as a sum of pos. int.

$$\prod_{j\geq 1} \frac{(1-q^{(\ell+1)j})^{\ell+1}}{1-q^j} = \sum_{n_1,\ldots,n_\ell \in \mathbb{Z}} q^{(\ell+1)\sum_{i=1}^{\ell} n_i^2 - (\ell+1)\sum_{i=1}^{\ell-1} n_i n_{i+1} - \sum_{i=1}^{\ell} n_i},$$

for $\ell = 1, 2, 3 \cdots$

Cook and Misra, 2005

Dynamical Systems
at NCSU

- Research Faculty

  - John Franke, Professor

    Research interest: Time dependent, discrete dynamical systems with applications to population biology.

  - Xiao-Biao Lin, Professor

    Research interest: Heteroclinic and periodic cycles, Riemann solutions of hyperbolic conservation laws, and Defermos regularization of a system of conservation laws.

  - Steve Schecter, Professor

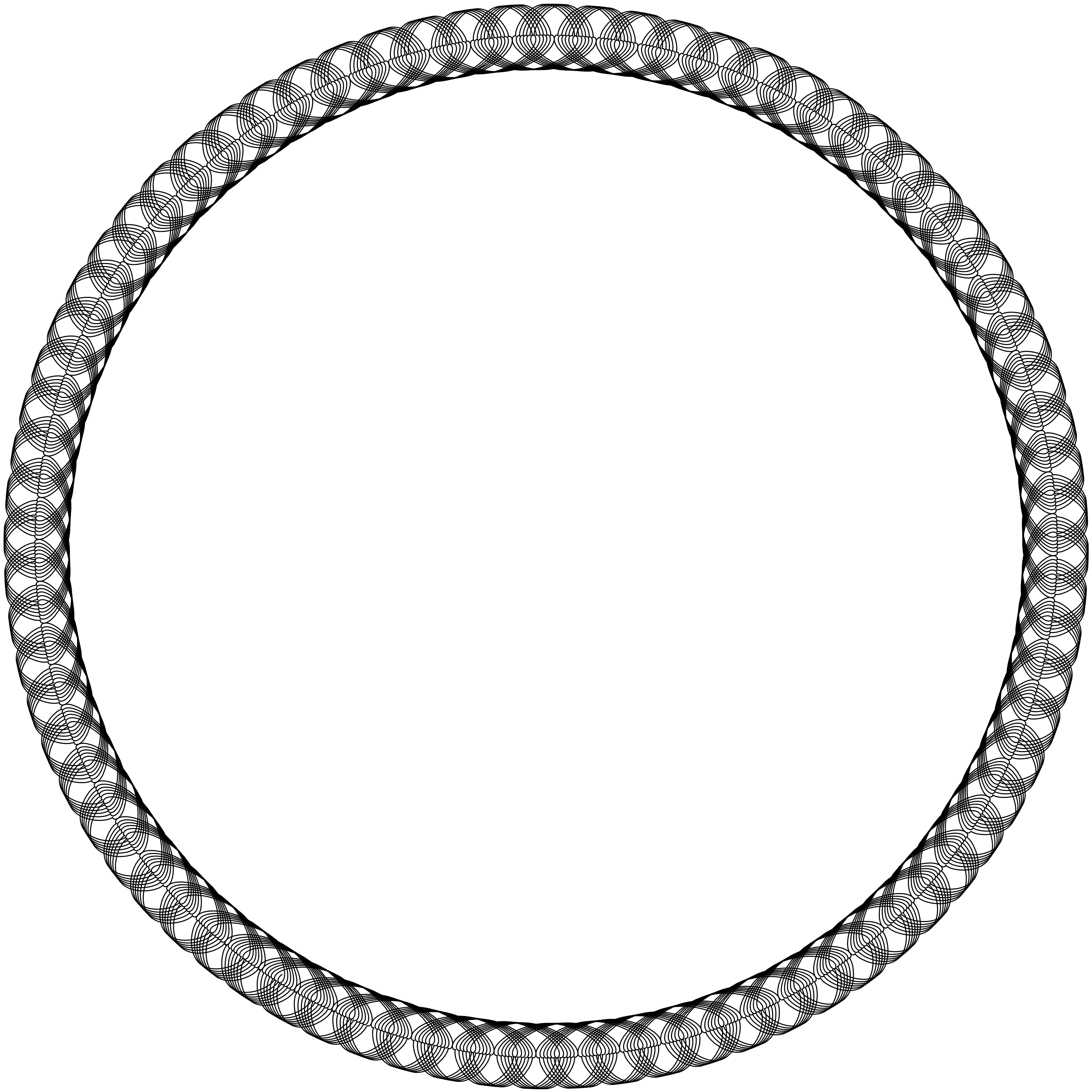
    Research interest: Shock waves, traveling waves and geometric singular perturbation theory.

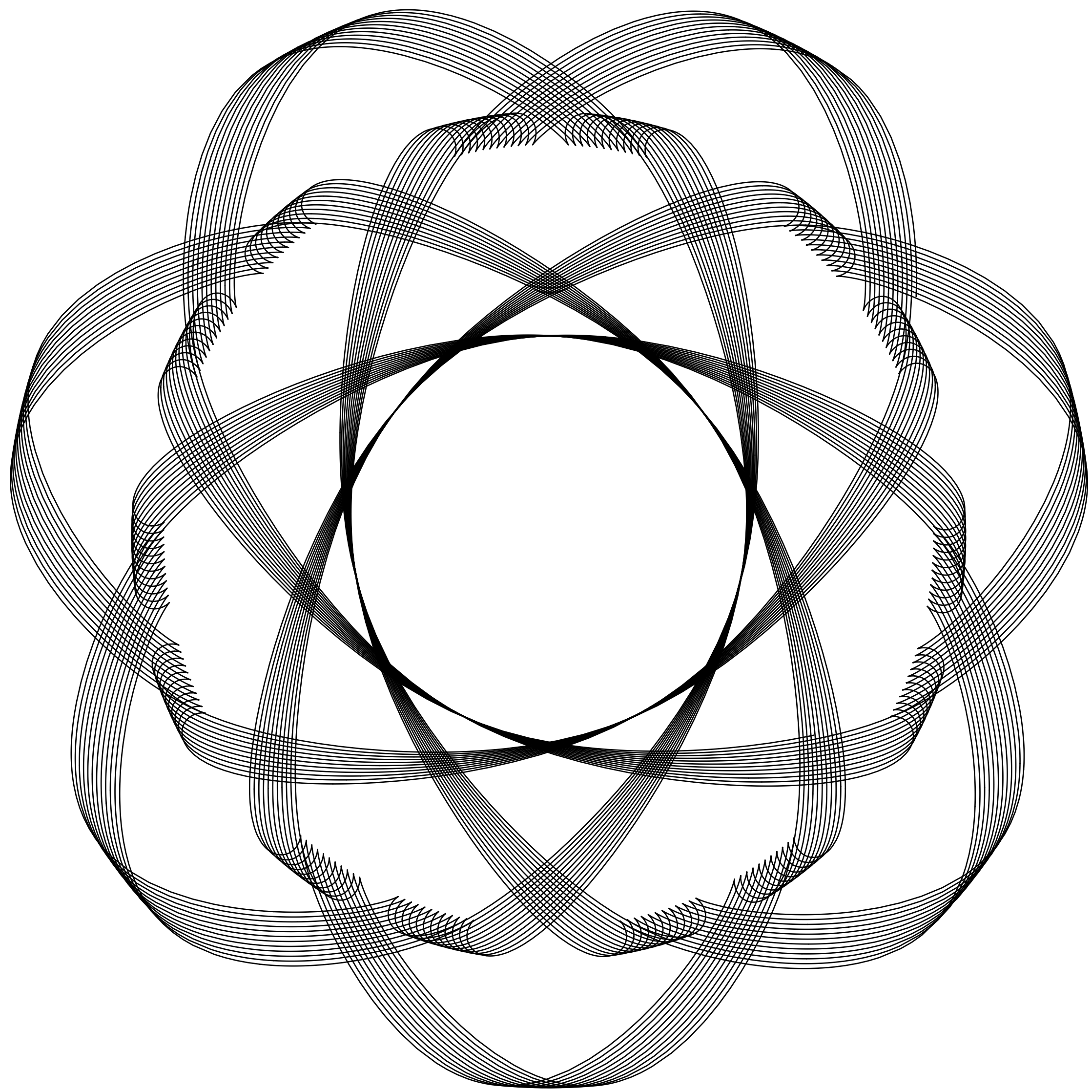
  - Jim Selgrade, Professor

    Research interest: (1) Dynamical behavior of solutions to differential and difference equation models in population biology and genetics, and (2) Modelling the estrogen (menstrual) cycle in humans.

  - Dmitry Zenkov, Assistant Professor

    Research interest: Integrability and stability of nonholonomic systems and dynamics of the discrete Chaplygin sleigh.





Symbolic Computation

Doing Mathematics by Computer

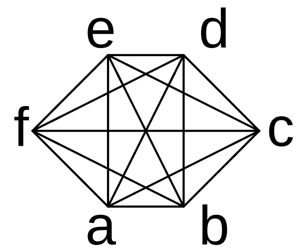
- RESEARCH FACULTY
  - Hoon Hong, Professor - Solving nonlinear constraints, real algebraic geometry, applications to computer aided design (CAD)
  - Erich Kaltofen, Professor - Symbolic linear algebra, symbolic-numeric computation.
  - Michael Singer, Professor - Symbolic analysis, algebraic theory of differential and difference equations.
  - Agnes Szanto, Assistant Professor -Polynomial system solving, symbolic-numeric computation.
  - Loek Helminck and Irina Kogan

- STUDENTS
  - 10 Ph.D. students graduated since 2001 hired by Antwerp, Duke, University of Illinois(Chicago), Lamar, Seton Hall, Wabash, Waterloo, NC Wesleyan, Cal State Bakersfield, Armstrong Atlantic
  - 7 Ph.D. students in progress

- OTHER ACTIVITY
  - Joint projects and meetings with groups in Argentina, France, China
  - Organized International Conferences (ISSAC, MEGA, ACA), Special Sessions at AMS meetings, Computer Algebra Day
  - Editorial Boards of J. Symb. Comp.; Alg. and Comp. in Math.; Applicable Algebra in Eng., Comm. and Comp.

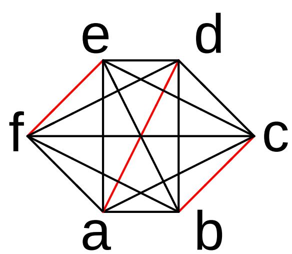
Closed form solutions

K_n = complete graph on n labelled vertices



How many maximal matchings in K_n? = a_n

Matching = set of edges with distinct endpoints



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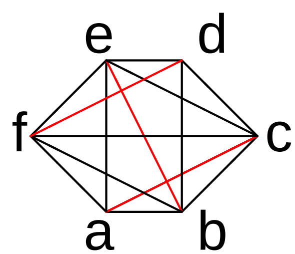

+ 13 others

$$a_{n+4} - (2n+5)a_{n+2} + (n+2)(n+1)a_n = 0$$

$$\implies \quad a_n = \begin{cases} \prod_{i=1}^{n/2} \quad (2i-1) & \text{if } n \text{ is even} \\ \prod_{i=1}^{(n+1)/2} (2i-1) & \text{if } n \text{ is odd} \end{cases}$$

$$a_2 = 1, \;\; a_3 = 3, \;\; a_4 = 3, \;\; a_5 = 15 \;\; , a_6 = 15, \;\; a_7 = 105, \;\; \ldots$$

$$a_{n+k} + \alpha_{k-1}(n)a_{n+k-1} + \ldots + \alpha_0(n)a_n = 0$$

$\alpha_i(n) =$ a rational function of n

Express a_n in "finite terms"

Example:

$$a_n + na_{n-1} + na_n = 0$$

$$\Rightarrow \quad a_n = Au_n + Bv_n, \quad A, B \in \mathbb{C}$$

$$u_n = (-1)^n(n-2)$$

$$v_n = (-1)^n\left(\frac{-(n-1)!}{n-2} + (n-2)\left[\sum_{k=3}^{n-1}(k-1)!\frac{k^3-6k^2+9k-3}{(k-1)^2(k-2)^2}\right]\right)$$

"Finite terms" =:

- Rational Funictions
- $\prod \cdots$
- $\sum \cdots$
- Interlacings $(2, 4, 8, \ldots), (3, 9, 27, \ldots) \Rightarrow (2, 3, 4, 9, 8, 27, \ldots)$

Algorithm - Hendricks and Singer