Math 132.4

Lesson 4: Property Appraisal and Linear Estimation

4.1 Application.

In this lesson we consider the appraisal of a home based on five criteria: age, number of bath rooms, number of bed rooms, heated square feet and general condition. Several homes in a similar location have been sold recently and numerical values for all five criteria and sales prices are known. A home which is not yet sold will have given numerical values for the five criteria. Based on the past sales we want to estimate the value of the home which is for sale.

4.2 Math Model.

Generally, if any of the criteria is larger, then the value of the home should increase. For appraisals with "comparable" homes, one can expect the value of the for sale home to be related in a linear way to the other homes:

Value = b + m1*x1 + m2*x2 + m3*x3 + m4*x4 + m5*x5 where

b, m1,…, m5 are to be found and

x1 = age

x2 = bath rooms

x3 = bed rooms

x4 = area and

x5 = condition.

We will use LINEST to find b, m1,…, m5 from the known data for the sold homes.

4.3 Method of Solution.

4.3.1 Table Method.

The following data has been collected about the recent sales of similar homes. One could attempt to "interpolate" between the homes with similar criteria (x1,…, x5) numbers and those of the for sale home. Since there are five criteria, this can become some what subjective!

age = x1	#bath = x2	#br = x3	area = x4	condition = x5	value = y
10	2.5	3	2000	9	160000
12	2	5	2500	8	17000
15	2	4	1900	6	130000
8	3	3	1600	9	150000
11	2.5	4	2300	7	160000
9	2	3	2000	8	150000
10	2	4	2400	9	180000

Table: Data for Seven Past Home Sales

4.3.2 Graph Method.

In the previous lesson one could try to visualize the data by a 3D graph where x1 and x2 correspond to x and y directions and the price is in the z direction. However, the appraisal problem has five variables and one value

$$\text{value} = b + m1*x1 + m2*x2 + m3*x3 + m4*x4 + m5*x5.$$

Since it is difficult to visualize in 6 dimensions, the graph method is not an applicable method.

4.3.3 Algebra Method.

In order to find b, m1,…, m5, we must define what we mean by a solution. Here we can gain some insight from the "least squares" approach to the problems in previous lessons. We want to choose b, m1,…., m5 so that the sale values V_j in the sixth column in the above table with j = 1,…,7 will be "close" to the computed values

$$CVj = b + m1*x1_j + m2*x2_j + m3*x3_j + m4*x4_j + m5*x5_j$$

where the $x1_j$,…, $x5_j$ are in columns 1 to 5 in the above table. "Close" will mean to choose b, m1,…, m5 so that the following **least squares function** will be a minimum

$$f(b, m1,\ldots,m5) = (V_1 - CV_1)^2 + (V_2 - CV_2)^2 + (V_3 - CV_3)^2 + (V_4 - CV_4)^2$$

$$+ (V_5 - CV_5)^2 + (V_6 - CV_6)^2 + (V_7 - CV_7)^2 \ .$$

There are two methods for doing this. One is a variation on "completing the square" and the other method is a variation on "derivatives." Presently, we be content with letting Excel do this computation.

4.4 Implementation.

The following spreadsheet segment uses LINEST to find b, m1,…,m5 for the above sales data.

age=x1	#bath=x2	#br=x3	area=x4	cond=x5	value=y	
10	2.5	3	2000	9	160000	
12	2	5	2200	8	170000	
15	2	4	1900	6	130000	
8	3	3	1600	9	150000	
11	2.5	4	2300	7	160000	
9	2	3	2000	8	150000	
10	2	4	2400	9	180000	
LINEST(f2:f8,a2:e8)						
m5 =	m4 =	m3 =	m2 =	m1 =	b =	
8827.1	35.90016	8082.088	4491.313	-1126.62	-15539.8	
Appraised Value = m1*x1 + m2*x2 + m3*x3 + m4*x4 + m5*x5 + b						
x5 =	x4 =	x3 =	x2 =	x1 =	one	
9	1900	4	2.5	9	1	
Appraised Value						
165531.4						

PART OF LINEST HELP FILE:

Uses the "least squares" method to calculate a straight line that best fits your data and returns an array that describes the line. The equation for the line is:

$$y = m_1*x1 + m_2*x2 + ... + b \text{ or } y = mx + b$$

where the dependent y-value is a function of the independent x-values. The m-values are coefficients corresponding to each x-value, and b is a constant value. Note that y, x, and m can be vectors. **The array that LINEST returns is $\{m_n,m_{n-1},...,m_1,b\}$.** LINEST can also return additional regression statistics.

LINEST(known_y's, known_x's)

Known_y's is the set of y-values you already know in the relationship y = mx + b.

If the array known_y's is in a single column, then each column of known_x's is interpreted as a separate variable.

Example 1. Slope and Y-intercept

LINEST({1,9,5,7},{0,4,2,3}) equals {2,1}, the slope = 2 and y-intercept = 1.

Example 2. Simple Linear Regression

Suppose a small business has sales of $3100, $4500, $4400, $5400, $7500, and $8100 during the first six months of the fiscal year. Assuming that the values are entered in the range B2:B7,respectively, you can use the following simple linear regression model to estimate sales for the ninth month.

SUM(LINEST(B2:B7)*{9,1}) equals SUM({1000,2000}*{9,1}) equals $11,000

In general, SUM({m,b}*{x,1}) equals mx + b, the estimated y-value for a given x-value. You can also use the TREND function.

Instructions for Home Appraisal:

Step 1. Enter the sales data for the seven sold homes in rows 2 to 8.

Step 2. LINEST must have at least two arrays for input data. This first is the input of selling values data (indicated by y in the Excel help file) and the second input data is the columns of the sales data (indicated by x in the Excel help file). Locate these in the table in step 1.

Step 3. LINEST will have output of six variables m5, …, m1 and b and in this order. Select six adjacent cells and enter "=LINEST(y,x)" where x and y are the cells for the data and in this order. In the above segment of a spreadsheet y = f2:f8 and x = a2:e8.

Step 4. In order to execute LINEST on WIN you must enter it by simultaneously pressing

ctrl + shift + enter

4.5 Assessment.

The value of a home is very closely related to the location. Therefore, a linear model for one location may vary a great deal from another location. Also, the choice of a linear model should be limited to homes with similar data. The use of certain data does not include all characteristics of a home.

4.6 Possible Homework.

1. Verify the computations for the above appraisal problem.

2. Use the above to estimate the value of a similar home whose criteria are:

   age is 13 year, has 2.5 bath rooms, has 3 bed rooms, has 2000 sq. ft. and general condition is 8.

3. Use LINEST to find a linear function that fits the following data for 7 recently sold homes.

age=x1	#bath=x2	#br=x3	area=x4	cond=x5	value=y
8	2.5	3	2000	9	160000
9	2	5	2200	8	140000
13	2	4	1900	6	130000
8	2.5	3	1600	9	150000
11	2.5	4	2300	7	160000
9	2	3	2000	8	150000
7	3	4	2400	9	160000

4. Use the function in problem 3 to appraise the following home:

   age is 12 year, has 2 bath rooms, has 4 bed rooms, has 2400 sq. ft. and general condition is 7.