Math 132.3

Lesson 3: Price Data for Two Markets and Linear Estimation

3.1 Application.

This lesson is a continuation of lesson two, where we considered the profit from one market with price per unit function that depended only on the sales in a single market. In this lesson we will consider the two market problem where the price per unit functions depend on both sales in each market. The math model for the price data will be linear in each variable for the sales. However, the resulting algebraic problem is a little more difficult, and so, we will use the Excel command called LINEST for linear estimation of the price data. We will indicate how this linear price function of the two sales variables can be used to determine the maximum profit.

3.2 Math Model.

Let the number of units sold in the country A be x, and the number of units sold in country B be y. Consider the two market problem with the following price per unit functions:

$$p_A = 97 - x/10 - y/200 \text{ and}$$

$$p_B = 83 - y/20 - x/100.$$

The new terms are y/200 and x/100, and they represent some "interaction" between the two markets. The coefficients 97, -1/10 and -1/200 for the price in country A must be determined from previous sales data, and this can be done by using the Excel command LINEST. This is a generalization of the Excel commands for trendlines because here price per unit will have the form

$$p = b + m1*x1 + m2*x2 \text{ where}$$

x1 = x, x2 = y are past sales and b, m1 and m2 are to be determined. This is the math model for the price data.

Once the two price functions are found, the profit as a function of two variables can be determined. Then the revenue is

$$R(x,y) = (97 - x/10 - y/200)x + (83 - y/20 - x/100)y.$$

Suppose the cost is

$$C(x,y) = 20{,}000 + 3x + 3y$$

where 20,000 is the initial cost and 3 is the cost to produce each item.

Therefore, the math model for the profit is

$$P(x,y) = R(x,y) - C(x,y)$$

$$= (97 - x/10 - y/200)x + (83 - y/20 - x/100)y$$

$$- (20{,}000 + 3x + 3y)$$

$$= 94x - x^2/10 + 80y - y^2/20 - (3/200)xy - 20{,}000.$$

Methods for finding the x and y that make this profit a maximum will be discussed in later lessons. Now, we will focus on finding the linear price function of two variables.

3.3 Method of Solution.

3.3.1 Table Method.

The x = x1 sales are in the first column, and the y = x2 sales are in the second column. The price per unit at these sales levels is recorded in the third column. Inspection of the table suggests that the price may be estimated by a linear function of the sales x1 and x2. Usually there is much more data than below.

sales=x1	sales=x2	priceA
10	5	45
12	4	44
13	3	43
15	3	39

Table: Data for priceA(x1,x2)

3.3.2 Graph Method.

One could try to visualize the data by a 3D graph where x1 and x2 correspond to x and y directions, and the price is in the z direction. By lowering or raising, and tilting in the x or y directions a plane can be “fit” to the data. However, unlike the problem with just one market, this is not really very practical in estimating the coefficients in the linear model with two variables

$$p = b + m1*x1 + m2*x2.$$

3.3.3 Algebra Method.

In order to find b, m1 and m2, we must define what we mean by a solution. Here we can gain some insight from the "least squares" approach to the problem with just one market. We want to choose b, m1 and m2 so that the measured price p_j in the third column in the above table with j = 1,…,4 will be "close" to the computed prices

$$b + m1*x1_j + m2*x2_j$$

where the $x1_j$ and the $x2_j$ are in the first and second column in the above table. "Close" will mean to choose b, m1 and m2 so that the following **least squares function** will be a minimum

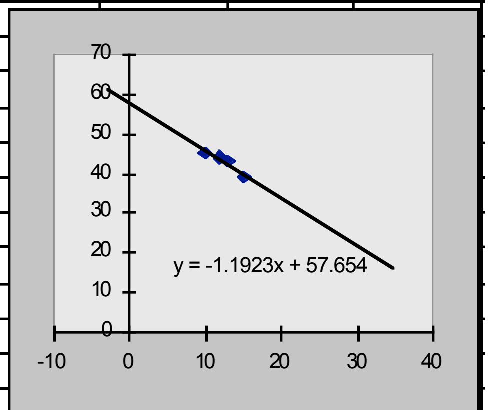
$$\begin{aligned} f(b, m1,m2) = &(p_1 - (b + m1*x1_1 + m2*x2_1))^2 \\ &+ (p_2 - (b + m1*x1_2 + m2*x2_2))^2 \\ &+ (p_3 - (b + m1*x1_3 + m2*x2_3))^2 \\ &+ (p_4 - (b + m1*x1_4 + m2*x2_4))^2. \end{aligned}$$

There are two methods for doing this. One is a variation on "completing the square" and the other method is a variation on "derivatives." In previous lessons we did both these for the one parameter least square problem. Presently, we will be content with letting Excel do this computation.

3.4 Implementation.

In the following segment of a spreadsheet there are two problems. The first is to determine the linear price function of just one variable price = b + m*x. Here we have solved this in two ways in Excel: use "trendline" and use LINEST. The second problem is for the price data with two markets.

sales=x	price=y	
10	45	
12	44	
13	43	
15	39	
LINEST(B2:B5,A2:A5)		
-1.19231	57.65385	
price(x) = -1.19231*x +		
33.80765		
sales=x1	sales=x2	priceA
10	5	45
12	4	44
13	3	43
15	3	39
LINEST(C18:C21,A18:B21)		
-1.90909	-2	74.90909
priceA(x1 ,x2) = -1.90909*x2 - 2*x1 +		
29.18206		



The linest command or program has two sets of cells as inputs and one set of cells as output. In the second example above the input cells are c18:c21 and a18:b21, and the output cells are a24:c24. The linest command is executed by typing in the selected output cells “=linest(c18:c21,a18:b21)”.

PART OF LINEST HELP FILE:

Uses the "least squares" method to calculate a straight line that best fits your data and returns an array that describes the line. The equation for the line is:

$$y = m_1*x1 + m_2*x2 + ... + b \text{ or } y = mx + b$$

where the dependent y-value is a function of the independent x-values. The m-values are coefficients corresponding to each x-value, and b is a constant value. Note that y, x, and m can be vectors. **The array that LINEST returns is $\{m_n,m_{n-1},\ldots,m_1,b\}$.** LINEST can also return additional regression statistics.

LINEST(known_y's, known_x's)

Known_y's is the set of y-values you already know in the relationship y = mx + b.

If the array known_y's is in a single column, then each column of known_x's is interpreted as a separate variable.

Example 1. Slope and Y-intercept

LINEST({1,9,5,7},{0,4,2,3}) equals {2,1}, the slope = 2 and y-intercept = 1.

Example 2. Simple Linear Regression

Suppose a small business has sales of $3100, $4500, $4400, $5400, $7500, and $8100 during the first six months of the fiscal year. Assuming that the values are entered in the range B2:B7,respectively, you can use the following simple linear regression model to estimate sales for the ninth month.

SUM(LINEST(B2:B7)*{9,1}) equals SUM({1000,2000}*{9,1}) equals $11,000

In general, SUM({m,b}*{x,1}) equals mx + b, the estimated y-value for a given x-value. You can also use the TREND function.

Instructions for Price in Two Markets:

Step 1. Enter the sales data for the first market in column a, sales data for the second market in column b and the price data in column c.

Step 2. LINEST must have at least two arrays for input data. This first is the input of measured price data (indicated by y in the Excel help file) and the second input data is the columns of the sales data (indicated by x in the Excel help file). Locate these in the table in step 1.

Step 3. LINEST will have output of three variable m2, m1 and b and in this order. Select three adjacent cells and enter "=LINEST(y,x)" where x and y are the cells for the data and in this order. In the above segment of a spreadsheet y = c18:c21 and x = a18:b21

Step 4. In order to execute LINEST on WIN, you must enter it by simultaneously pressing ctrl + shift + enter

3.5 Assessment.

There can be many more than one or two markets for a product. As soon as one goes to more than two markets, the table and graphing become much more of a burden. However, the algebraic method still works very nicely when one has good computing tools.

Not all data will look linear in both variables, and therefore, the choice of a linear model would not be appropriate. Consider the following variation on the above two market data where the price as a function of x2, for x1 = 10, decreases like a parabola.

Sales = x1	**Sales = x2**	**Price**
10	5	84
12	5	82
14	5	79
10	6	81
10	7	73
10	8	56

Table: Nonlinear Data

3.6 Possible Homework.

1. Verify the computations for the above two market problem.
2. Consider the profit function in section 4.2. Modify it by using the new price function for country A.
3. Use LINEST to find a linear price function for the following data for three markets.

$$\text{Price} = b + m1*x1 + m2*x2 + m3*x3$$

sales=x1	sales=x2	sales=x3	priceA
10	5	7	45
12	6	6	43
13	6	5	41
15	7	5	41
16	8	5	38
18	9	4	37

4. Find the price functions in problem three if the sales price varies by plus or minus 10%.