Math 132.2

Lesson 2: Price Data and Trendlines

2.1 Application.

In the previous lesson we used the price per suit as a given linear function of the number of suits sold, $p(x) = -.05x + 80$. The slope, -.05, and the intercept, 80, were based on observations by the retailer. The objective of this lesson is to discuss methods for finding these numerical values based on a few previous observations. The accepted method for doing this cannot be fully developed until the end of this course; however, we will assume the y intercept is known and this will simplify the problem.

Assume the initial price of a suit is $80. When x = 10, 20, and 30 suits have been sold, the price per suit dropped to $79, $77 and $76, respectively. From these observations we would like to be able to predict the price of a suit for larger x. This will effect the revenue and the profit. If the price drops too low, the profit may be negative!

2.2 Math Model.

When the sales went to x = 10, the price dropped $1. When the sales went from x = 10 to x = 20, the price dropped by $2. When the sales went from x = 20 to x = 30, the price dropped by $1. Would it be reasonable to speculate that the price will drop from $1 to $2 if the sales increased from x = 30 to x = 40? Often sales of similar suits have followed this pattern. So, a mathematical model for the price per suit as a function of x would be a linear equation

$$p(x) = mx + c$$

where m and c are to be determined. Here we will fix c = 80 to be the initial price.

2.3 Method of Solution.

The first two approaches are preliminary to the "least squares" method which is an algebraic interpretation the graphical approach. The table method illustrates the need to organize the initial observations in a systematic way. In fact, in many similar problems there are massive amounts of past observations.

2.3.1 Table Method.

The data will be organized into two columns; the first column will have the values of x, and the second column will have the corresponding price per suits. We have also included a third column which have the possible slopes, [change in price]/[change in sales].

x	price	change in price/change in sales
0	80	
10	79	(79-80)/(10-0)
20	77	(77-79)/(20-10)
30	76	(76-77)/(30-20)

Table: Possible Slopes

The possible slopes are in the third column and are -1/10, -2/10 and -1/10. Perhaps one should let the m in our model be the average of these m = (-.1 + -.2 + -.1)/3 = -.1333.

Then our model for the price function would be

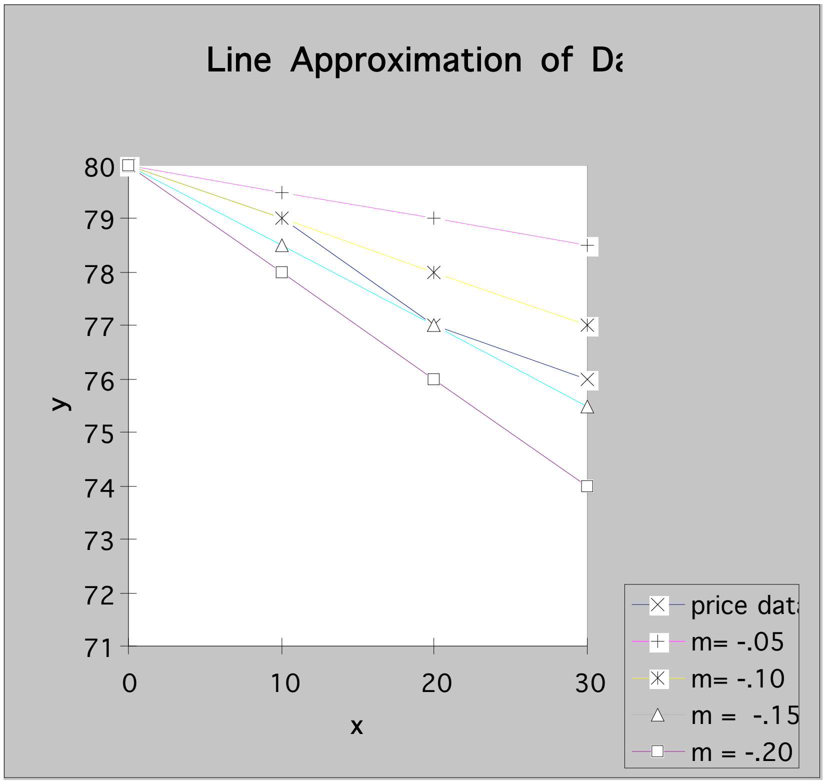
$$p(x) = -.1333x + 80.$$

From here one could predict the price when x = 100 suites

$$p(x) = -.1333(100) + 80. = 67.67.$$

2.3.2 Graph Method.

A second approach is to graph the data along with a variety of straight lines with different slopes. In the following graph we have used c = 80 and m = -.05, -.10, -.15 and -.20. By visual inspection of the graph it appears that the "best" choice for m is some where between m = -.10 and m = -.15. This is consistent with the above method.



2.3.3 Algebra Method.

Both the above methods are some what intuitive, and the outcome may be dependent on the individual who is using those methods. What is needed is a more precise measure of how "close" a line is to the given data. One measure is the "sum of the squares" function. The following table has four columns. The first two columns have the above sales and price data. The third column has a computed price function with m unknown. The fourth column has the square of difference in the price data and the computed price. The sixth row of the fourth column has the definition of the "sum of the squares" function.

x	price	mx + 80	(price-(mx+80))^2
0	80	m0 +80	0
10	79	m10 +80	(-1-m10)^2
20	77	m20 +80	(-3-m20)^2

30	76	m30 +80	(-4-m30)^2
			sum of sq.= (-1-m10)^2 + (-3-m20)^2 + (-4-m30)^2

Table: Sum of the Squares

Definition. Consider the above price data and the math model for the price per suit, p(x) = mx +80. The following function of the unknown slope, m, is called the "**sum of the squares**" function:

$$f(m) = (-1 - m10)^2 + (-3 - m20)^2 + (-4 - m30)^2.$$

Definition. The m that makes f(m) the smallest will generate a function p(x) = m x + 80 which we will call a "**one parameter least squares**" approximation to the above price data.

A method for finding the m that makes f(m) the smallest is based on the fact that f(m) is a parabola which opens upwards and the "completion of the square" technique.

$$f(m) = (-1 - m10)^2 + (-3 - m20)^2 + (-4 - m30)^2$$

$$= 1 +2m10 + 100m^2 + 9 + 2(3)m20 + 400m^2 + 16 +2(4)m30 + 900m^2$$

$$= 26 + 380m + 1400m^2$$

$$= 1400 (m^2 + 2(190/1400)m + 26/1400)$$

$$= 1400 (m^2 + 2(19/140)m + (19/140)^2 - (19/140)^2 + 26/1400)$$

$$= 1400 ((m + 19/140)^2 + (26/1400 - 19/140)).$$

Thus, f(m) will be a minimum when m = -19/140. That is, m = -19/140 = -.1357 and the "one parameter least squares" approximation to the above suit data is p(x) = -.1357x + 80. So, for this approximation the predicted price for the 100^{th} suit is p(100) = -.1357(100) + 80. = 66.43.

Another method for finding the minimum of f(m) is the "derivative technique". Here we must

compute the derivative of f(m),

set it equal to zero and

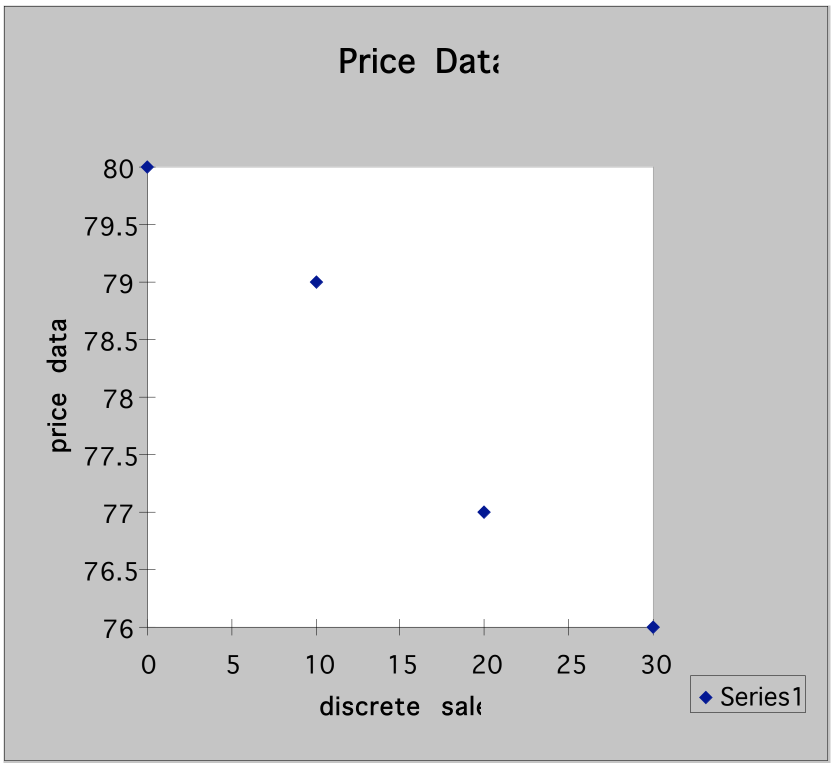
solve for m.

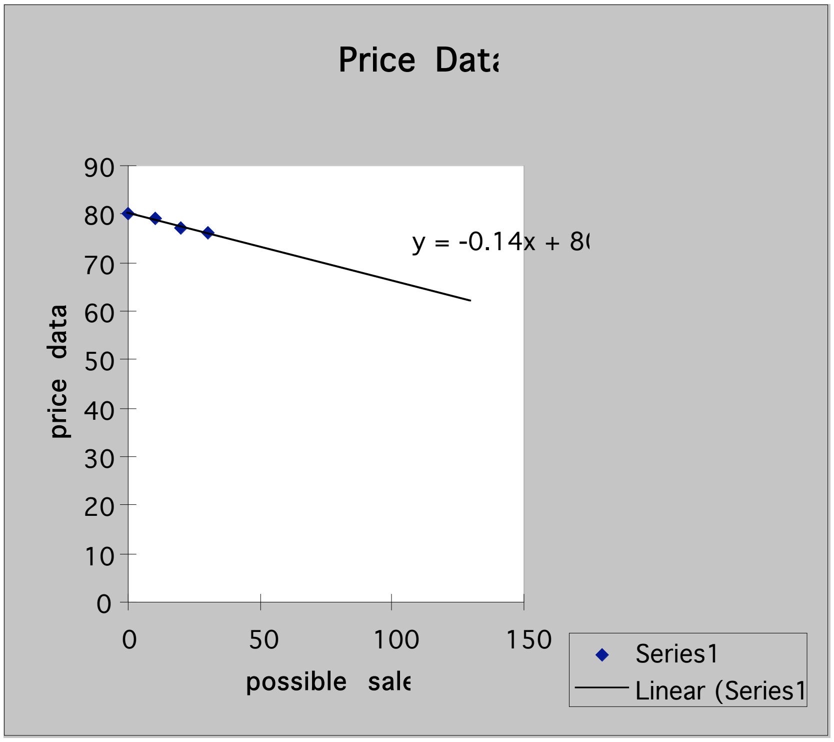
For the above f(m) this is

$$f'(m) = 1400 \cdot 2(m + 19/140)^{2-1} + 0 = 0 \text{ and so } m = -19/140.$$

2.4 Implementation.

The following is an implementation on the Excel spreadsheet where both the m and the c are unknowns; this is called the **"least squares"** function. For the suit data it is p(x) = -.14x + 80.1, and so, the predicted price a for the sales of the 100^{th} suit is p(100) = -.14(100) + 80.1 = 66.1.





Instructions for Graph of Discrete Data:

Step 1. Open a spreadsheet and enter the data in columns a and b.

Step 2. Select the above cells and click the “chart wizard” button.

Step 3. Choose "scatter."

Step 4. Follow the instructions.

Instructions for Generating the "Least Squares" Function:

Step 1. Use the above "scatter" chart and double click on a data point.

Step 2. Under the “chart” menu choose “add trendline.”

Step 3. Choose "linear."

Step 4. Under the option tab choose "display equation", "forecast ahead = 100" and OK.

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2.5 Assessment.

In the above suit sales data the price was generally in a downward trend of about .1 dollar for every suit sold. The precise nature of this trend is debatable. Indeed, we were able to generate four different price functions. Which one is the "best" model? Perhaps, the only real test is whether or not they predict the price for any additional sales. Generally, the least squares method as implemented in the Excel spreadsheet is accepted.

Not all data will always look like a straight line. For the suit data it would be unreasonable for the price to be extrapolated to zero! So, for large range of sales one can expect the price to level off to some positive value. In the Excel spreadsheet there are many other choice of "trendlines" such as polynomials or power functions. The choice of the "trendline" is part of the modeling process which must be based on previous similar applications.

Another possible problem is with the accuracy of the data. What happen to the predicted prices if the data has an error of 10% or 20%? What happens to the predicted profits?

2.6 Possible Homework.

1. Verify that the above "least squares" function $p(x) = -.14x + 80.1$ agrees with your computing tool.

   (a). Use it with the cost function $C(x) = 20x + 1{,}000$ to find $P(x)$.

   (b). Find the break even sales levels, that is, where $P(x) = 0$.

   (c). Find the sales level where the profit will be a maximum.

2. Suppose the suit sales data is modified as follows:

x	**price per suit**
0	80
10	78
20	75
30	73

   (a). Use all four methods to find a price function $p(x) = mx + c$ that approximates the above suit data.

   (b). Use all four price functions to predict the price of suit when $x = 100$.

   (c). Use the price per unit from the "least square" function to find the profit function when the cost function is $C(x) = 20x + 10{,}000$.

(d). Graph the profit function and find the two break even points and the sales where the profit will be a maximum.

(e). Repeat parts (c) and (d) when the cost per suit increases from \$20, \$25 to \$30.

3. Consider a new model of car whose sales are as follows:

x	**price per car**
0	18,000
10	17,500
20	17,200
30	17,000

(a). Repeat problem two parts (c) and (d) with C(x) = 6,000x + 1,000,000.

(b). Suppose the price data is off by plus or minus 10%. How does this vary your calculations in part (a)?