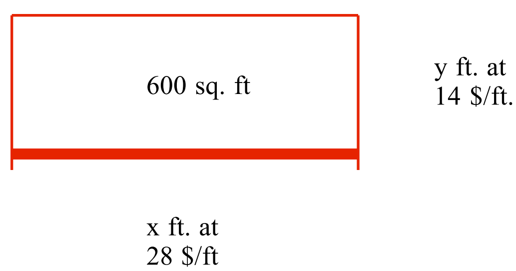
# Math 132.10

# Lesson 10: Minimum Cost of a Display Area and Derivatives

## 10.1 Application.

A rectangular display area is to have 600 sq. ft. One side is to be constructed of cement and costs 28 \$/ft. The other three sides are constructed of wood and cost 14 \$/ft. Find the dimensions of the display area so that the cost is a minimum.




## 10.2 Math Model.

The formulation of the math model is as follows. The cost function has two parts: the cost of the concrete side and the cost of the three wood sides.

[total cost] = [cost of concrete side] + [cost of the wood sides]

$$= 28\,x \qquad + \qquad 14\,(y + x + y)$$

$$= 42\,x + 28\,y.$$

The constraint equation is a result of the requirement of 600 sq. ft.

$$600 = x\,y.$$

So, $y = 600/x$, and we can put this into the cost function. This will give us a function of just one variable for the cost of the display area

$$C(x) = 42\,x + 16800/x.$$

# 10.3 Method of Solution.

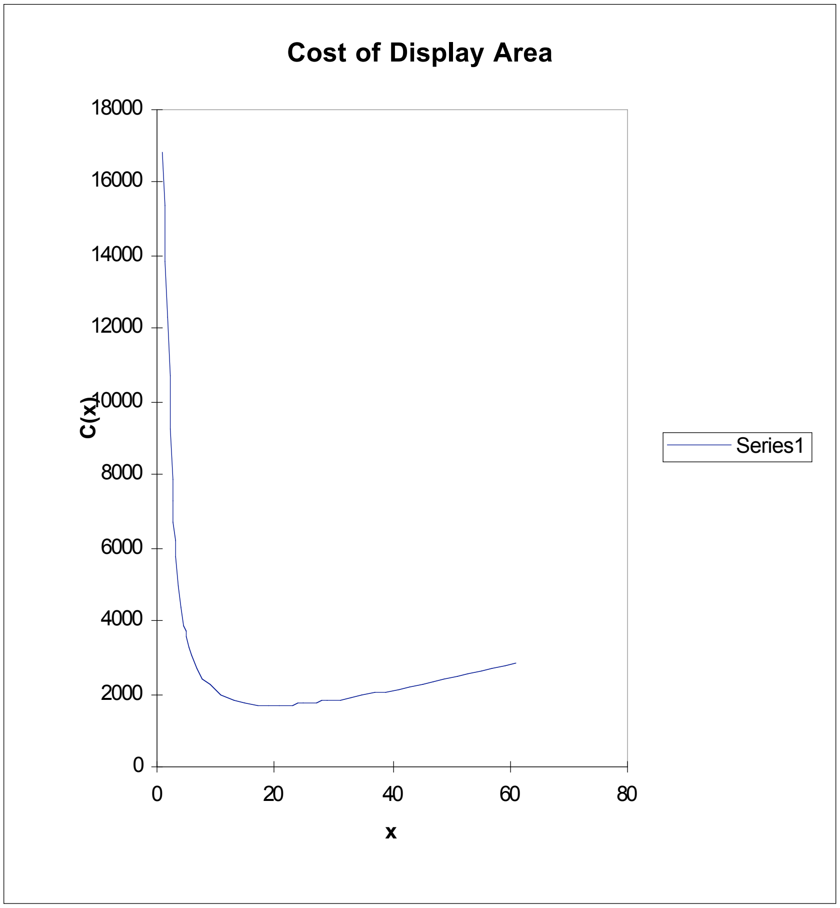
## 10.3.1 Table Method.

| x | Cost |
|---|---|
| 1 | 16842 |
| 3 | 5726 |
| 5 | 3570 |
| 7 | 2694 |
| 9 | 2244.667 |
| 11 | 1989.273 |
| 13 | 1838.308 |
| 15 | 1750 |
| 17 | 1702.235 |
| 19 | 1682.211 |
| 21 | 1682 |
| 23 | 1696.435 |
| 25 | 1722 |
| 27 | 1756.222 |
| 29 | 1797.31 |
| 31 | 1843.935 |
| 33 | 1895.091 |
| 35 | 1950 |
| 37 | 2008.054 |
| 39 | 2068.769 |
| 41 | 2131.756 |
| 43 | 2196.698 |
| 45 | 2263.333 |
| 47 | 2331.447 |
| 49 | 2400.857 |
| 51 | 2471.412 |
| 53 | 2542.981 |
| 55 | 2615.455 |
| 57 | 2688.737 |
| 59 | 2762.746 |
| 61 | 2837.41 |

**Table: Size and Cost**

By inspection of the table and the graph of $C(x) = 42x + 16800/x$, the minimum cost must be about 1680 and is attained when x is near 20. Both the table and graph methods have the same problem: it is difficult to determine if one has all the relevant parts of the table or graph. Since the function is not as well known as a straight line or as a parabola, we do not know if there are additional "wiggles" in the graph. If there are "wiggles", then there must be local maximum or minimum points. That is, there must be additional points where the derivative is zero.

## 10.3.2 Graph Method.




## 10.3.3 Algebra Method.

By the derivative rules one can easily compute the derivative of the cost function $C(x) = 42\ x + 16800/x$:

$$C'(x) = 42\ (x)' + 16800\ (x^{-1})'$$

$$= 42\ (1) + 16800\ ((-1)\ x^{-1-1})$$

$$= 42 - 16800/x^2.$$

Setting the derivative equal to zero gives

$$0 = 42 - 1680/x^2,$$

$$16800 = 42x^2 \text{ and } x^2 = 400.$$

So, x must equal plus or minus 20. The minus 20 has no meaning in our problem, and therefore, only the positive x = 20 is where the minimum cost can be attained. From the table/graph we know the minimum cost is C(20) = 1680.

### 10.3.3 Algebra Method with Maple.

We can have Maple do all the previous work for us. There is nothing here we haven't already done in lesson 1, so the purpose of this part of the lesson is largely going to be review. The reader should recall from lesson 1 on maximizing profit how to define a function. We shall define the cost function above:

```
[> C:=x->42*x+16800/x;
```

To minimize the cost, we take the derivative:

```
[> dC:=D(C);
```

and then solve for when C' = 0:

```
[> solve(dC(x)=0);
```

20, -20

Just as before, this tells us that the cost is minimized when x equals 20 or minus 20, and as before, we choose x = 20. To find this minimum cost, we evaluate the function C at t=20:

```
[> C(20);
```

1680

This agrees with the results of the last section.

## 10.4 Implementation.

**Instructions for the Graph of the Cost Function:**

Step 1. Open a new sheet and enter "x" and "Cost" in cells a1:b1.

Step 2. Create a "series" for relevant values of x in column a

Step 3. In cell b2 enter the formula for the cost "= 42*a2 + 16800/a2".

Step 4. Create a "series" in column b for the cost function with changing x = a2, a3,..... Do this by selecting cell b2, moving the cursor to the lower right corner and dragging down column b.

Step 5. Select the relevant parts of this table and use the "chart wizard" to graph the cost function.

# 10.5 Assessment.

Often the minimum/maximum problems will have more constraints on the variable. For example, the above display might be required to have at least a certain length on each of its sides. This forces one to place inequality constants on the variable x. Then each of the three methods must be modified.

# 10.6 Possible Homework.

1. Verify the table and graph for the cost of the above display problem.
2. Consider a new display area which must have 600 $ft^2$ . Suppose the concrete side will cost 32 \$/ft. and the wood sides will cost 16 \$/ft.
   (a). Find the constraint and cost functions.
   (b). Use all three methods to find the minimum cost.
3. Suppose in problem two the concrete cost varies from 24, 28, 32 to 36 \$/ft. How does this change the minimum cost?
4. A rectangular area is to be fenced in with 500 ft. of fence. One side can use the existing building as retainer.
   (a). Find the constraint function and the objective function.
   (b). Use all three methods to find the dimensions of the area so that the area is a maximum.