The Geometry and Analysis of Dynamical Systems

Schedule for Friday, February 22, 2008

Feb. 22, 2008			Rm. #
1.15 p.m. - 1.30 p.m.		Welcome Remarks	335
1.30 p.m - 2.00 p.m.	Yingfei Yi	Quasi-periodic breathers in Hamiltonian networks	335
2.00 p.m. - 2.30 p.m	Carmen Chicone	An introduction to smoothed particle hydrodynamics	335
2.30 p.m. - 3.00 p.m.	Margaret Beck	Nonlinear stability of time-periodic viscous shocks	335
3.00 p.m. - 3.30 p.m.	Tasso Kaper	Wave speeds in R-D equations with cut-offs	335
3.30 p.m. - 4.00 p.m.		Tea Break	
4.00 p.m. - 5.00 p.m.	Bjorn Sandstede	Snakes and ladders	307
6.30 p.m. - ...	Banquet		

Schedule for Saturday, February 23, 2008

Feb. 23, 2008		Room 307
8.30 a.m. - 9.00 a.m.	Todd Kapitula	On the spectrum associated with periodic waves for generalized KdV equations
9.00 a.m - 9.30 a.m.	Weishi Liu	Current-Voltage relations of ion channels via Poisson-Nernst-Planck systems
9.30 a.m - 10.00 a.m.	Anna Ghazaryan	Nonlinear stability of fronts in a combustion model
10.00 a.m - 10.30 a.m.		Tea Break
10.30 a.m - 11.00 a.m.	Jack Hale	A general class of evolutionary equations
11.00 a.m - 11.30 a.m.	Kening Lu	Lyapunov exponents and invariant manifolds for random dynamical systems in a Banach space
11.30 a.m - 12.00 p.m.	Martin Wechselberger	Folded saddle-nodes: where canards meet Hopf (and Shilnikov)
12.00 p.m - 1.30 p.m.		Lunch
1.30 p.m - 2.00 p.m.	Kris Jenssen	Hyperbolic conservation laws with prescribed eigencurves
2.00 p.m. - 2.30 p.m.	Wenxian Shen	Variational principle for spatial spread and propagation speeds in time almost and space periodic KPP models
2.30 p.m. - 3.00 p.m.	Kevin Zumbrun	Numerical stability analysis of shock and detonation waves
3.00 p.m. - 3.30 p.m.	David Schaeffer	The little house on the hill: will it stay there when the rains come?
3.30 p.m. - 4.00 p.m.		Tea Break
4.00 p.m. - 4.30 p.m.	Chongchun Zeng	Invariant manifold of dynamic spike solutions to a singular parabolic equation
4.30 p.m. - 5.00 p.m.	Constantine Dafermos	BV solutions of the p-system with frictional damping
7.00pm- ..		Party

Abstracts in the order of appearance in the program

Yingfei Yi
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Quasi-periodic breathers in Hamiltonian networks

Abstract: Hamiltonian networks form an important class of infinite dimensional Hamiltonian systems arising in solid state physics, cell biology, and many other areas of science and technology. They also arise naturally in the discretization of Hamiltonian PDEs but the physical interest in Hamiltonian networks mainly lies in dynamics which are far away from those of Hamiltonian PDEs. Among interesting dynamics of a Hamiltonian network, of physical importance is a robust coherent structure known as breathers or quasi-periodic breathers which are self-localized, time periodic or quasi-periodic solutions. In this lecture, several models of Hamiltonian networks of long-range, weakly coupled anharmonic oscillators will be considered. It will be shown that corresponding to any fixed number of sites in such a Hamiltonian network, there is a positive Lebesgue measure set of linear stable, quasi-periodic breathers with the number of oscillating frequencies equal to the number of excited sites.

Carmen Chicone
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An introduction to smoothed particle hydrodynamics

Abstract: Smoothed particle hydrodynamics is a Lagrangian numerical method for approximating the equations of motion of continuum mechanics; in particular, hydrodynamics. The subject was introduced in the 1970s by astrophysicists. It has since developed into a useful numerical method that has been successfully applied to simulate many phenomena in continuum mechanics. The underlying ideas for the method will be introduced, some problems for mathematical research will be suggested, and an application to a free boundary problem—water flowing over a horizontal plate—will be presented.

Margaret Beck
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Nonlinear stability of time-periodic viscous shocks

Abstract: Time-period viscous shocks, for example created through a Hopf bifurcation, are shown to be nonlinearly stable. The result is obtained by developing a contour integral formulation of the linear evolution, associated to the time-periodic linearized operator. This allows for detailed pointwise estimates and isolation of the neutral directions corresponding to space and time translation. A Gronwall-type argument is then used to prove nonlinear stability of the time-periodic shocks.

Tasso Kaper
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Wave speeds in R-D equations with cut-offs

Abstract: The Fisher-Kolmogorov-Petrowskii-Piscounov (FKPP) equation with cut-off was introduced in [E. Brunet and B. Derrida, Shift in the velocity of a front due to a cutoff, Phys. Rev. E, 56, 2597–2604 (1997)] to model N-particle systems in which concentrations less than epsilon= 1/N are not attainable. It was conjectured that the cut-off function, which sets the reaction terms to zero if the concentration is below the small threshold epsilon, introduces a substantial shift in the propagation speed of the corresponding traveling waves. In this talk, we prove the conjecture of Brunet and Derrida, and extend this result to a more general family of scalar reaction-diffusion equations with cut-off. The main mathematical techniques used in our proof are geometric singular perturbation theory and the blow-up method, which lead naturally to the identification of the reasons for the logarithmic dependence of the critical wave speed on epsilon, as well as for the universality of the corresponding leading order coefficient. This is joint work with Freddy Dumortier and Nikola Popovic.

Björn Sandstede
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Snakes and ladders

Abstract: I will discuss localized 1D and 2D structures such as hexagon patches, localized radial target patterns, and localized 1D rolls, which arise as stationary solutions of PDEs. All these solutions exhibit snaking: in parameter space, the localized states lie on a vertical wiggly bifurcation curve so that the spatial extent of the underlying periodic pattern, such as hexagons or rolls, increases as we move up along the snaking curve. I will give an overview of recent analytical and numerical works in which this phenomenon is investigated.

Todd Kapitula
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On the spectrum associated with periodic waves for generalized KdV equations

Abstract: In joint work with Mariana Haragus, I consider the problem of determining the spectrum for the linearization of an infinite-dimensional Hamiltonian system about a spatially periodic traveling wave. By using a Bloch-wave decomposition, we recast the problem as determining the point spectra for a family of operators $\mathcal{J}_\gamma\mathcal{L}_\gamma$, where $\mathcal{J}_\gamma$ is skew-symmetric with bounded inverse and $\mathcal{L}_\gamma$ is symmetric with compact inverse. Our main result relates the number of unstable eigenvalues of the operator $\mathcal{J}_\gamma\mathcal{L}_\gamma$ to the number of negative eigenvalues of the symmetric operator $\mathcal{L}_\gamma$. The compactness of the resolvent operators allows to greatly simplify the proofs, as compared to those of similar results for linearizations about localized waves. The theory is applied to a study of the spectra associated with periodic solutions to the generalized Korteweg-de Vries equation with power nonlinearity.

Weishi Liu

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Current-Voltage relations of ion channels via Poisson-Nernst-Planck systems

Abstract: An important characterization of ion channel properties is the current-voltage (I-V) relation. A basic model for mathematical study of I-V relations is the Poisson- Nernst-Planck (PNP) systems. In this talk, we will discuss some recent results on the multiplicity and spatial behavior of solutions of PNP systems and on a specific nonlinear feature of the I-V relation in a special case.

Anna Ghazaryan

University of North Carolina-Chapel Hill & North Carolina State University

Nonlinear stability of fronts in a combustion model

Abstract: I will discuss nonlinear stability of fronts in a model for combustion of high density liquid fuels. When the fuel thermal diffusivity is small, the spectral information is not definitive. Carefully chosen exponential weights stabilize the spectrum of the front, but cause the nonlinearity to lose its smoothness. The issue is resolved by analyzing a system that consists of equations for the original variables coupled to equations for the weighted variables. It is shown that the front in a co-moving frame is nonlinearly stable in an exponentially weighted norm if initial perturbations are small in the original and weighted norms.

Jack Hale

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A general class of evolutionary equations

Abstract: Motivated by the fact that a dynamical system may evolve through an observable quantity rather than the state of the system, a general class of evolutionary equations is defined which includes standard ordinary and partial differential equations as well as partial functional differential equations of retarded and neutral type. We give some details about a general theory of the evolution of temperature in a solid material as well as transmission lines on a circle with dynamic boundary conditions. When the abstract equation is linear, there are interesting unsolved spectral problems.

Kening Lu

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Lyapunov Exponents and invariant manifolds for random dynamical systems in a Banach space

Abstract: We study the Lyapunov exponents and their associated invariant subspaces for infinite dimensional random dynamical systems in a Banach space, which are generated by, for example, stochastic or random partial differential equations. We prove a multiplicative ergodic theorem. Then, we use this theorem to establish the stable and unstable manifold theorem for nonuniformly hyperbolic random invariant sets.

Martin Wechselberger
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Folded saddle-nodes: where canards meet Hopf (and Shilnikov)

Abstract: Folded saddle-nodes (FSN) occur in generic one parameter families of singularly perturbed systems with two slow variables. These singularities are the organizing centers for two main delay phenomena: canards and delayed Hopf bifurcations. We will combine techniques from canard analysis, geometric singular perturbation theory and blow-up, and from delayed Hopf bifurcation analysis, complex time paths, to understand the flow near FSN. If we combine the local flow near FSN with an appropriate global return mechanism then this leads to complex oscillatory patterns known as mixed mode oscillations (MMO). Such MMO could be (partly) interpreted as a novel Shilnikov type behaviour.

Kris Jenssen
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Hyperbolic conservation laws with prescribed eigencurves

Abstract: Motivated by recent examples of singular behavior in systems of hyperbolic conservation laws in one space dimension, we consider the following problem: Given n vector fields $R_1, ..., R_n$ in $\mathbb{R}^n$; determine if there exists an $n \times n$-system of hyperbolic conservation laws with a flux function whose Jacobian has the R_i as right eigenvectors.

We formulate this as a problem for the eigenvalues of the system. For $n \geq 3$ this typically gives an over-determined system of PDEs for the eigenvalues to which we apply Cartan-Kä hler theory. Several concrete examples are also given.

This is joint work with Irina Kogan (NCSU).

Wenxian Shen
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Variational principle for spatial spread and propagation speeds in time almost and space periodic KPP models

Abstract. The present talk is concerned with spatial spread and propagation speeds in time almost periodic and space periodic KPP models. A notion of spatial spread (propagation) speed for a time almost periodic and space periodic KPP model is introduced for the first time, which extends the concept of the spatial spread (propagation) speed for time independent or periodic KPP models and can be applied to more general time dependent KPP models. By applying the recently developed principal spectrum theory for general time dependent linear parabolic equations, a variational principle for the spatial spread (propagation) speed is established. Based on the variational principle, the influence of time and space variation on the spread speed is discussed. It is shown that the time and space variation speeds up the spatial spread. The obtained results provide basic foundations for the study of KPP models subject to various spatial and temporal variations.

Kevin Zumbrun
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Numerical stability analysis of shock and detonation waves.

Abstract: We discuss recent progress in numerical determination of stability, both at the level of algorithms/efficiency, and also singular perturbation/strategies for global stability analysis.

David G. Schaeffer
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The little house on the hill: will it stay there when the rains come?

Abstract. This talk analyzes a model for a layer of soil slipping down a hill. Such phenomena precede the occurrence of catastrophic landslides. Depending on parameters, the model admits multiple solutions. We shall focus on two of these: (i) slow, steady creep of the top layer of soil down a slope and (ii) stick-slip motion–i.e., brief intervals of rapid sliding alternating with comparatively long periods during which the top layer does not move at all. The main point of the talk is to understand the transition between the two types of solutions, which is mediated by a Hopf bifurcation.

The bifurcation is driven by ground water pressure in the soil, which is governed by the heat equation. Mathematically, it is interesting to see periodic solutions emerging from this dissipative equations.

Chongchun Zeng
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Invariant manifold of dynamic spike solutions to a singular parabolic equation

Abstract: Consider a nonlinear parabolic equation $u_t = \epsilon^2 \Delta u - u + f(u)$ on a smooth bounded domain $\Omega \subset \mathbb{R}^n$ with the Neumann boundary condition. In the past years, there had been extensive studies on steady spike solutions. Here a spike solution u is one which is almost equal to zero everywhere except on a ball of radius $O(\epsilon)$ where the $u = O(1)$. In this talk, we show that there exist dynamic spike solutions which maintain the spike profile for all $t \in \mathbb{R}$ with the spike moving on $\partial\Omega$.

Constantine Dafermos
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BV solutions of the p-system with frictional damping