

MATHEMATICS DEPARTMENT
North Carolina State University

DIFFERENTIAL EQUATIONS SEMINAR

Wednesday, September 13, 2006
3:00 p.m. 330 Harrelson Hall

Xiao-Biao Lin

Department of Mathematics
North Carolina State University

“The Gearhart-Prüss Theorem and linear
stability for Riemann solutions of
conservation laws”

In this talk, we will review Hille-Yosida Theory, the Paley-Wiener Theorem, and the Gearhart-Prüss Theorem on the asymptotic behavior of semigroups. We then consider the spectral and linear stability of Riemann solutions with multiple Lax shocks for systems of conservation laws $u_\tau + f(u)_\xi = 0$. Using the self-similar change of variables $x = \frac{\xi}{\tau}$, $t = \ln \tau$, Riemann solutions become stationary solutions of the system $u_t + (Df(u) - xI)u_x = 0$. In the space of $O((1 + |x|)^{-\eta})$ functions, we show that if $\Re \lambda > -\eta$, then λ is either an eigenvalue or a resolvent point. Eigenvalues of the linearized system are zeros of the determinant of a transcendental matrix. Some vertical lines in the complex plane are *resonance lines*, on which the determinant becomes arbitrarily small but nonzero. A C^0 -semigroup is constructed. Using the Gearhart-Prüss Theorem, we show that the solutions are $O(e^{\gamma t})$ if γ is greater than the largest real part of the eigenvalues and the resonance lines. We study examples of Riemann solutions having two or three Lax shocks.

Graduate students are invited to attend.