MATHEMATICS DEPARTMENT
North Carolina State University

DIFFERENTIAL EQUATIONS SEMINAR

Wednesday, October 12, 2005
3:00 p.m. 330 Harrelson Hall

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"A first-order PDE describing particle-size segregation in granular avalanches"

Particle-size segregation in avalanches occurs through shearing within the granular flow. In such a flow, large particles migrate upwards, their vacated spaces being filled by smaller particles. It is widely acknowledged that size segregation plays a crucial role in many aspects of avalanche flow, such as setting the speed of the front, and generating wave patterns, both surface waves and transverse waves observed at the leading edge. In a recent paper, Gray and Thornton proposed a simple model to capture this segregation, based on conservation of mass for two-phase flow and basic mixture theory. The equation is a scalar conservation law in two space variables and time, but with variable coefficients corresponding to the spatially dependent velocity in shear flow. In this talk, I describe initial-boundary-value problems for this equation, and show numerical simulations. In simple circumstances, the problem can be solved explicitly, by combining basic multidimensional solutions (shocks and rarefaction waves). Interfaces with large particles below small are physically unstable, a property that is explained using the theory of shock wave stability. Unstable interfaces give rise to mixing zones, which we track numerically; solutions of a Riemann problem capture the essential features of the mixing zone as an evolving combination of shocks and rarefaction waves.

This talk will appeal to students who would like to see an application of nonlinear first-order scalar PDE.