

MATHEMATICS DEPARTMENT
North Carolina State University

JOINT DIFFERENTIAL
EQUATIONS/NUMERICAL ANALYSIS SEMINAR

Wednesday, April 12, 2006
3:00 p.m. 330 Harrelson Hall

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“Theory of bicharacteristics and genuinely
multidimensional finite volume schemes for
hyperbolic balance laws”

It is our belief that the most satisfying methods for approximating evolutionary PDEs are based on approximating the corresponding evolutionary operator. We explain our methodology on the system of shallow water equations with source terms modeling the bottom topography, friction effects, and Coriolis forces. Results can be generalized to more complex systems of balance laws.

In this talk we first present the theory of bicharacteristics for multidimensional systems of hyperbolic balance laws. This leads to exact integral equations, which take all of the infinitely many directions of wave propagation explicitly into account. We would like to point out that in contrast to the Kirchoff formulae, which give time-explicit representations, the integral equations based on bicharacteristics are implicit in time.

In the second part of the lecture we show how these integral equations are used in the construction of very accurate finite volume methods, the so-called finite volume evolution Galerkin (FVEG) schemes. They combine the usually conflicting design objectives of using the conservation form and following the characteristics or bicharacteristics. In fact, instead of using approximate Riemann solvers, which are basically one-dimensional, we use suitable approximation of the integral equations, the so-called approximate evolution operators. Note that the source terms are approximated in such a way that stationary states will be preserved exactly. We derive a well-balanced approximation of the integral equations and prove that the FVEG scheme is well-balanced for stationary steady states as well as for steady jets in the rotational frame. Several illustrative examples confirm this property experimentally.

This research has been done in cooperation with Bill Morton (Bath and Oxford Universities), Gerald Warnecke (University of Magdeburg), Sebastian Noelle (RWTH Aachen), and Marcus Kraft (Hamburg University of Technology).