

MATHEMATICS DEPARTMENT  
North Carolina State University

## DIFFERENTIAL EQUATIONS SEMINAR

Wednesday, September 28, 2005  
3:00 p.m. 330 Harrelson Hall

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### “ $L^2$ semigroup and linear stability for Riemann solutions of conservation laws”

Riemann solutions for the systems of conservation laws  $u_\tau + f(u)_\xi = 0$  are self-similar solutions of the form  $u = u(\xi/\tau)$ . Using the change of variables  $x = \xi/\tau, t = \ln(\tau)$ , Riemann solutions become stationary solutions of the system  $u_t + (Df(u) - xI)u_x = 0$ . For the linear variational system around a Riemann solution with  $n$  Lax shocks, we introduce a semigroup in a Hilbert space with weighted  $L^2$  norm. We show that (A) the region  $\Re\lambda > -\eta$  consists of normal points only. (B) Eigenvalues of the linear system correspond to zeros of the determinant of a transcendental matrix. They lie on vertical lines in the complex plane. There can be *resonance values* where the response of the system to forcing terms can be arbitrarily large. Resonance values also lie on vertical lines in the complex plane. (C) Solutions of the linear system are  $O(e^{\gamma t})$  for any constant  $\gamma$  that is greater than the largest real parts of the eigenvalues and the coordinates of *resonance lines*. This work can be applied to the linear and nonlinear stability of Riemann solutions of conservation laws and to the stability of nearby solutions of the Dafermos regularization  $u_t + (Df(u) - xI)u_x = \epsilon u_{xx}$ .

**Graduate students are invited to attend.**

For questions, comments, and offers to talk, contact Steve Schecter, [schecter@math.ncsu.edu](mailto:schecter@math.ncsu.edu).  
Please visit the DE Seminar web page at [www.math.ncsu.edu/DE](http://www.math.ncsu.edu/DE).