

MATHEMATICS DEPARTMENT
North Carolina State University

COLLOQUIUM

Thursday, November 13, 2003
4:00 p.m. 201 Harrelson Hall

Mark Levi

Department of Mathematics
Pennsylvania State University

“Stability webs in Hill’s equations”

Hill’s equation $\ddot{x} + q(t)x = 0$, also known as the Schrödinger equation in one dimension, is ubiquitous in various fields, including mechanics, Hamiltonian dynamics, and completely integrable systems. Because of its significance, this equation has been extensively studied. In the last 25 years, Arnold and Keller and Weinstein found some new properties of resonance tongues for Hill’s equations—domains in parameter space that correspond to instability. This talk discusses some more recently discovered topological features of stability diagrams for Hill’s equations, which, in light of the long history, were quite unexpected. The phenomenon is related to properties of the symplectic group $SL(2, \mathbb{R})$, and it occurs away from the resonances that were studied earlier. A certain feature of the stability diagrams, first discovered numerically by Simó, turns out to rely on a property of trigonometric potentials: If a trigonometric potential of degree two has two wells, then the periods of oscillations of two particles in different wells but with the same energy are equal. We will give a short proof of this fact.

Graduate students are invited to attend.

Tea will be served in HA 243 at 3:30 p.m.