

You must show all your work.

1 (20 pts) Let T be the linear transformation on \mathbb{R}^2 that first reflects along the line $x_1 + x_2 = 0$ and then rotates counterclockwise by 45 degree. Find the matrix representation of T with respect to the basis $\{[1, 2], [2, 1]\}$.

2 (15 pts) Find the coordinate vector of the vector $(5, 5, -8)$ with respect the basis $(2, 1, -1), (1, 3, -3), (3, 4, -1)$.

3 (20 pts) Let $B = \{1 - 2x, x, x^2\}$ and $C = \{x - 1, x + 4, x^2 - 1\}$ be two bases in P_2 .

(a) Find the transition matrix from B to C.

(b) Find the coordinate vector for $x^2 - 3x + 4$ with respect to the basis B .

(c) Find the polynomial such that its coordinate vector with respect to the basis C is $(3, 1, 2)^T$.

4 (25 pts) Find a set of linearly independent eigenvectors for the matrix

$$\begin{pmatrix} 1 & 3 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

5 (20 pts) Let L be the linear transformation given by $L(f) = x^2 f'' - x f' - 4f$ on P_2 , and the basis $B = \{x^2 + x + 1, x + 1, 1\}$.

(a) Find the matrix representation of L with respect to B .

(b) Find all possible polynomials f such that $L(f) = 0$.

(c) What are the eigenvalues of L ?