TEST THREE, MA 305, DR. JING'S SECTION
OCTOBER 27, 2005. 3:00-4:15

Print Your Name: **Row:** **Seat number:**

1. (20 pts) Judge if the following statements are true (T) or false (F).

(a) For any matrix $A \in M_{mn}$ the map $T(x) = Ax$ defines a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

(b) Let a fixed $B \in M_{mn}$, then the map $T(A) = AB$ is a linear transformation from M_{nm} to M_{nn}.

(c) The map $f(x) = (x_1 - 2x_2 + 2, x_1 - x_3, 3x_1 + x_2)$ defines a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$.

(d) The equation $Ax = b$ is always solvable when A has full rank.

(e) If nullity$(A) = 5$, then nullity$(A^TA) = 5$.

(f) If rank$(A^TA) = 5$, then nullity$(A) = 6$.

(g) The normal equation $A^TAx = A^Tb$ always has a unique solution.

(h) If the vector v_3 is a linear combination of two linearly independent vectors $\{v_1, v_2\}$, then the Gram-Schmidt process on $\{v_1, v_2, v_3\}$ will give three non-zero orthonormal vectors.

(i) If T and S are linear transformations from V to V, then $2T + 3S$ is also a linear transformation. Here $2T + 3S$ is given by $(2T + 3S)(v) = 2T(v) + 3S(v)$.

(j) The map $T(f) = 3f - 2f'$ is a linear transformation on the space P_3 of polynomials of degree less than or equal to 3. (f' is the derivative of f)

2. (20 pts) Let T be the linear transformation of $\mathbb{R}^2$ that reflects about the line $y = x$ and then rotates 30 degress counterclockwise. Find the matrix A for T such that $T(x) = Ax, x \in \mathbb{R}^2$.

3. (20 pts) Solve the following least squares system

$$\begin{aligned} 5x_1 + 4x_2 &= 1 \\ x_1 - x_2 &= 2 \\ 2x_1 + 3x_2 &= -2 \\ 3x_1 - 5x_2 &= 0 \end{aligned}$$

4. (20 pts) Use the Gram-Schmidt process to find an orthonormal basis for $RS(A)$, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 0 \end{bmatrix}$$

5. (20 pts) Let $f(x) = (x_1 + x_2, 3x_1 - 4x_2, 3x_1 - x_2)^T$ be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$.

(a) Can you find all vectors $x \in \mathbb{R}^2$ such that $f(x) = (0, 0, 0)^T$.

(b) Find all vectors x such that $f(x) = (2, -1, 2)^T$.