MA305, Test Two, Dr. Jing Name ________________

10/04/2005, 3:00-4:15 Row Number ________________

You must show all your work except the multiple choices problems.

1 (20 pts) Judge if the following statements are true (T) or false (F).

(a) $S = \{(s+2, s, t) | s, t \quad \text{are real numbers}\}$ is a subspace of $\mathbb{R}^3$.

(b) $S = \{(x_1, x_2) \in \mathbb{R}^2 | x_1 + 3x_2 = 0\}$ is a subspace of $\mathbb{R}^2$.

(c) If S is a spanning set of a vector space V, and S is a subset of T, then T is also a spanning set of V.

(d) If A is a 3×3-matrix and $det(A) \neq 0$, then any vector in R^3 is in the column space of A.

(e) Any six vectors of P_4 are linearly dependent, here P_4 consists of polynomials with degree ≤ 4.

(f) If A and B have the same (reduced) echelon form, then there is a sequence of row operations to change A into B.

(g) The unit disk $D = \{(x, y) | x^2 + y^2 \leq 1\}$ is a subspace of $\mathbb{R}^2$.

(h) The set of all unit triangular 3×3 matrices of the form $\begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ b & c & 0 \end{pmatrix}$ is a subspace of $M_{3\times 3}$, where a, b, c are arbitrary.

(i) Any subspace of R^n can be realized as a column space of some matrix.

(j) If $A \to \cdots \to U$ gives the echelon form U for the matrix A under row operations, then the row subspace of A equals to the row subspace of U.

2 (25 pts) (a) Find the row subspace RS(A), column subspace CS(A), and the nullspace NS(A) for the following matrix A

$$A = \begin{bmatrix} 2 & -4 & 3 & 0 & 1 \\ -3 & 1 & 2 & 3 & 0 \\ 1 & -7 & 8 & 3 & 2 \\ 3 & -4 & 11 & 3 & 3 \end{bmatrix}$$

(b) What are the rank and nullity of A?

3 (25 pts) Determine if polynomial $p = 2x^2 - 11x + 32$ is a linear combination of the following three polynomials $q_1 = x^2 - 3x + 1, q_2 = x^2 - 2x - 5, q_3 = x^2 - 5x + 13$. If it is, find all possible coefficients c_1, c_2, c_3 such that $p = c_1q_1 + c_2q_2 + c_3q_3$.

4 (30 pts) Let $W = \{(x_1, x_2, x_3, x_4) | x_4 = x_1 + x_2, x_3 = x_1 - x_2\}$

(a) Find a matrix A such that $W = NS(A)$.

(b) Find a basis for W.

(c) Find a matrix B such that $W = CS(B)$